CHAPTER 14

Shafts

14.1 Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

Notes: 1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

14.2 Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

14.3 Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

14.4 Types of Shafts

The following two types of shafts are important from the subject point of view :

1. *Transmission shafts.* These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts.* These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

14.5 Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5 m, 6 m and 7 m.

14.6 Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

14.7 Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(*a*) 112 MPa for shafts without allowance for keyways.

(*b*) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$$\sigma_t = 0.6\ \sigma_{el} \text{ or } 0.36\ \sigma_u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

(*a*) 56 MPa for shafts without allowance for key ways.

(*b*) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el}) but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible shear stress,

$$\tau = 0.3\ \sigma_{el} \text{ or } 0.18\ \sigma_u, \text{ whichever is less.}$$

14.8 Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(*a*) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,

(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

14.9 Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \ldots(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre
= $d / 2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...}(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft
= d_i / d_o

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \text{...}(iv)$$

Shafts inside generators and motors are made to bear high torsional stresses.

From the equations ***(iii)*** or ***(iv)***, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\,R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : N = 200 r.p.m. ; P = 20 kW = 20 × 10^3 W; τ = 42 MPa = 42 N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ d^3 = 955 × 10^3 / 8.25 = 115 733 or d = 48.7 say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : P = 1 MW = 1 × 10^6 W ; N = 240 r.p.m. ; $T_{max} = 1.2\ T_{mean}$; τ = 60 MPa = 60 N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

∴ $$d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or $$d = 159.4 \text{ say } 160 \text{ mm}$$ **Ans.**

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

∴ $$d^3 = 955 \times 10^3 / 8.84 = 108\ 032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm}$$ **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\ (d_o)^3\ [1 - (0.5)^4] = 8.3\ (d_o)^3$$

∴ $$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\ 060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm}$$ **Ans.**

and $$d_i = 0.5\ d_o = 0.5 \times 50 = 25 \text{ mm}$$ **Ans.**

14.10 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \text{...}(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \qquad \text{...(where } k = d_i / d_o)$$

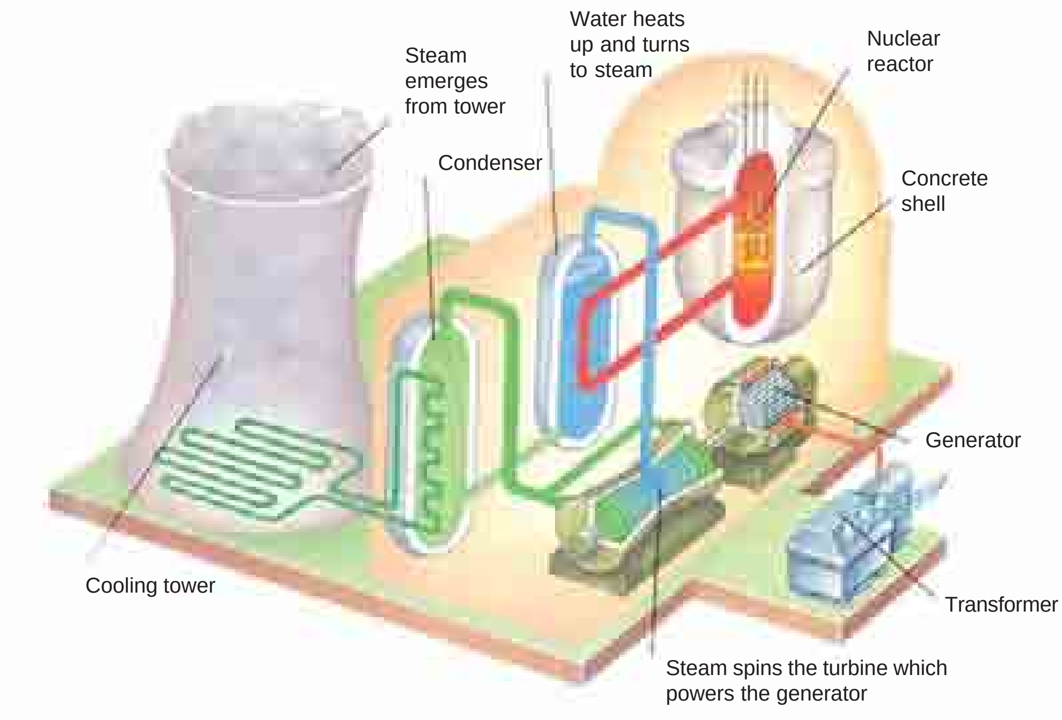
and $y = d_o / 2$

Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Note: We have already discussed in Art. 14.1 that the axles are used to transmit bending moment only. Thus, axles are designed on the basis of bending moment only, in the similar way as discussed above.



In a neuclear power plant, stearm is generated using the heat of nuclear reactions. Remaining function of steam turbines and generators is same as in theraml power plants.

Example 14.4. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50$ kN $= 50 \times 10^3$ N ; $L = 100$ mm ; $x = 1.4$ m ; $\sigma_b = 100$ MPa $= 100$ N/mm^2

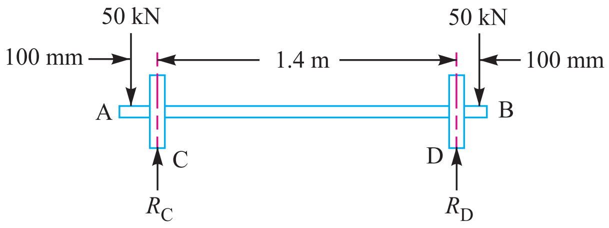


Fig. 14.1

The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm}$$ **Ans.**

14.11 Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

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* The maximum B.M. may be obtained as follows :

$R_C = R_D = 50$ kN $= 50 \times 10^3$ N

B.M. at *A*, $M_A = 0$

B.M. at *C*, $M_C = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

B.M. at *D*, $M_D = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6$ N-mm

B.M. at *B*, $M_B = 0$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation **(*i*)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi\,d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations **(*ii*)** and **(*v*)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and $$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : M = 3000 N-m = 3 × 10^6 N-mm ; T = 10 000 N-m = 10 × 10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

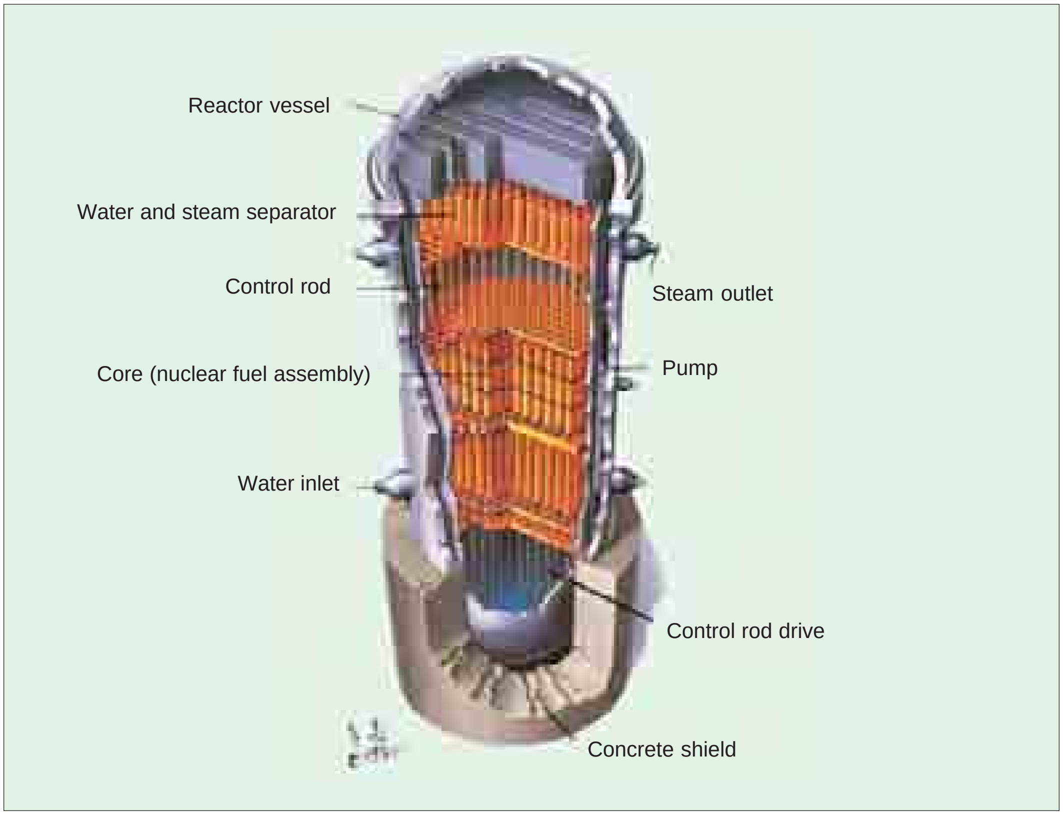
According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$



Nuclear Reactor

Note : This picture is given as additional information and is not a direct example of the current chapter.

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$

$$= \tfrac{1}{2}\,(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \quad \text{or} \quad d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Example 14.6. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.

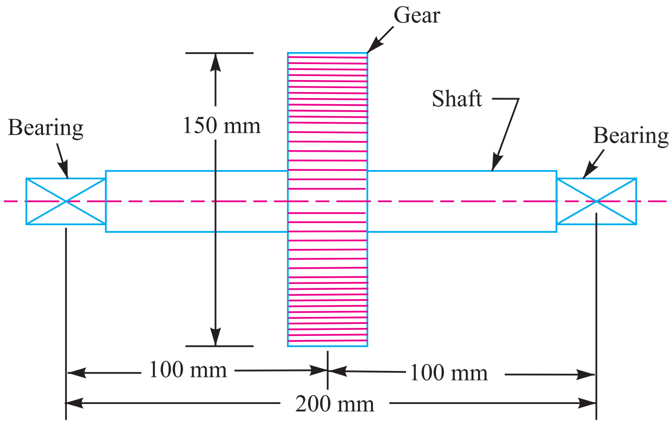


Fig. 14.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

∴ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20°} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or d = 32 say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

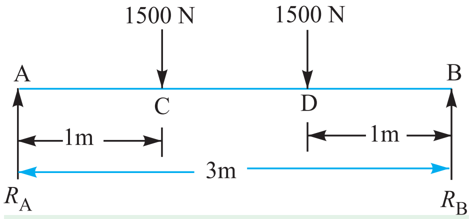


Fig. 14.3

$\therefore$ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\ d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2)$$

$\therefore$ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or d = 66.8 say 70 mm **Ans.**

Example 14.8. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be* *overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

Solution . Given : D = 1.5 m or R = 0.75 m; T_1 = 5.4 kN = 5400 N ; T_2 = 1.8 kN = 1800 N ; L = 400 mm ; τ = 42 MPa = 42 N/mm^2

A line shaft with a pulley is shown in Fig 14.4.

We know that torque transmitted by the shaft,

$$T = (T_1 - T_2)\,R = (5400 - 1800)\,0.75 = 2700 \text{ N-m}$$
$$= 2700 \times 10^3 \text{ N-mm}$$

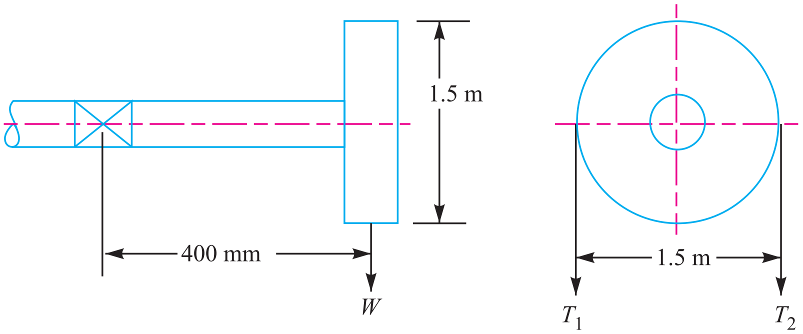


Fig. 14.4

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

∴ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$
$$= 3950 \times 10^3 \text{ N-mm}$$

Steel shaft

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 3950 \times 10^3/8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Example 14.9. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*

Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm^2 ; $\tau = 42$ MPa $= 42$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley C = 2250 N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and
$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

= 0

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

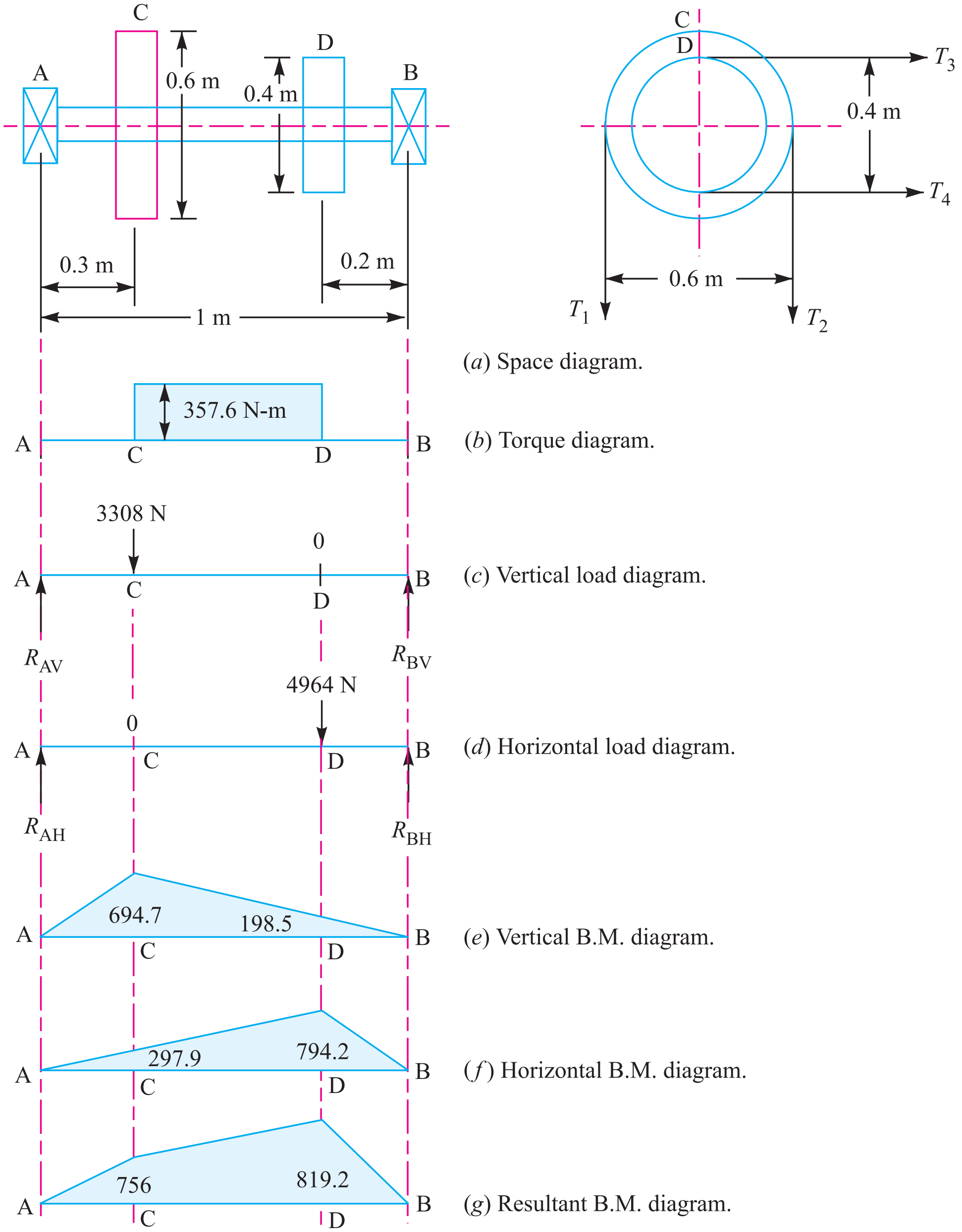
Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...(}i\text{)}$$

We know that
$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...(}ii\text{)}$$



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

Fig. 14.5

From equations **(*i*)** and **(*ii*)**, we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

∴ Horizontal load acting on the shaft at *D*,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at *C* = 0

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at *D*, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about *A*,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at *D*, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (*g*).

We see that bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

∴ $$d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \quad \text{or} \quad d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$
$$= \tfrac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$$\therefore \qquad d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \quad \text{or} \quad d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm}$$ **Ans.**

Example 14.10. *A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.*

Solution. Given : AB = 800 mm ; $\alpha_C = 20°$; D_C = 600 mm or R_C = 300 mm ; AC = 200 mm ; D_D = 700 mm or R_D = 350 mm ; DB = 250 mm ; $\theta = 180° = \pi$ rad ; W = 2000 N ; T_1 = 3000 N ; $T_1/T_2 = 3$; τ = 40 MPa = 40 N/mm²

The space diagram of the shaft is shown in Fig. 14.6 (*a*).

We know that the torque acting on the shaft at D,

$$T = (T_1 - T_2)\, R_D = T_1 \left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000 \left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because\ T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 14.6 (*b*).

Assuming that the torque at D is equal to the torque at C, therefore the tangential force acting on the gear C,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.7. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

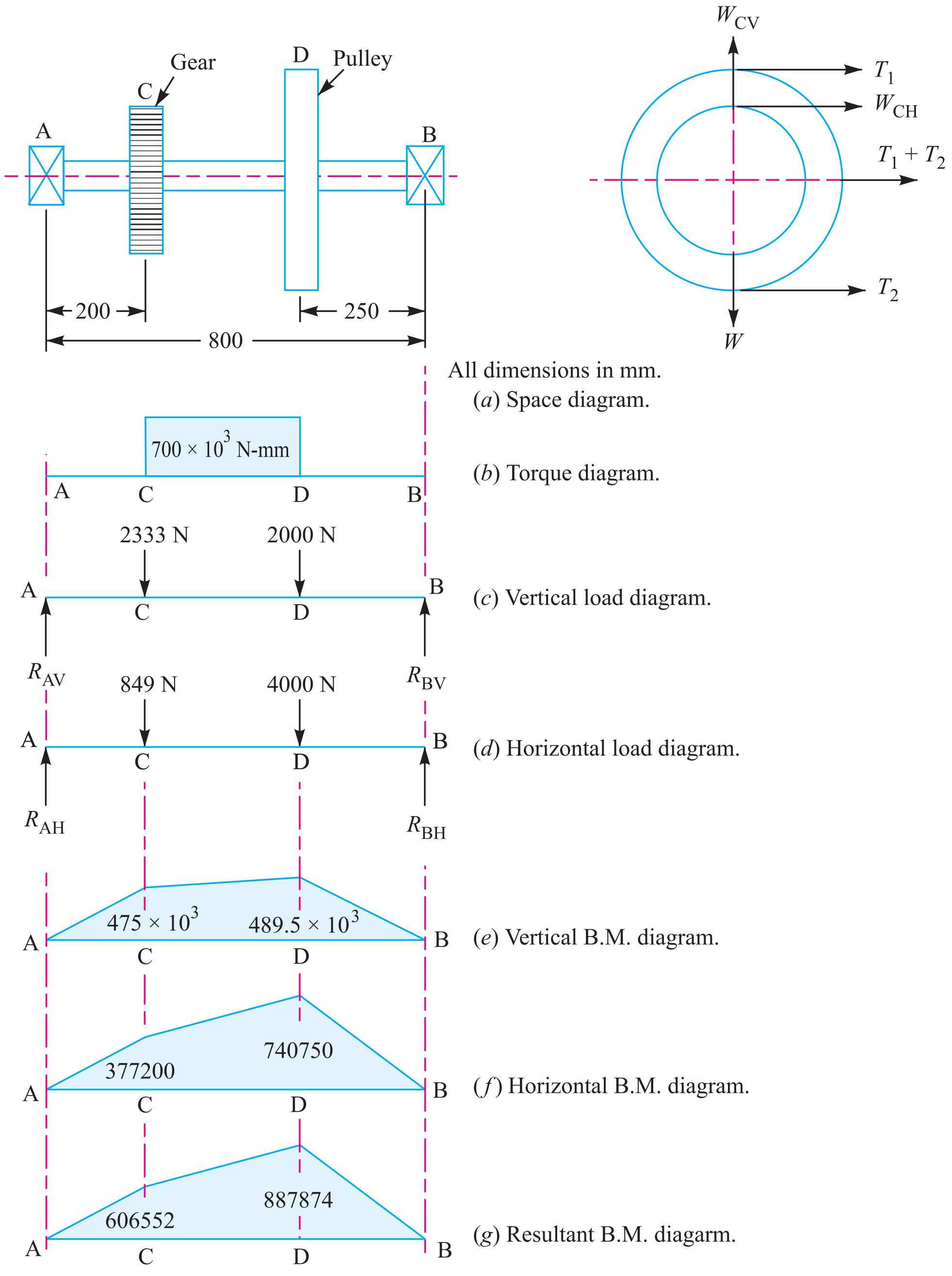
and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and T_1 = 3000 N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$


Camshaft



All dimensions in mm.

(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagarm.

Fig. 14.6

∴ Horizontal load acting on the shaft at *D*,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at *D*,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at *C* and *D* is shown in Fig. 14.6 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at *C* and *D*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

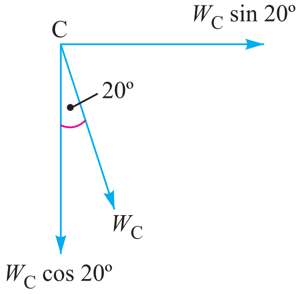


Fig. 14.7

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 800 = 2000\ (800 - 250) + 2333 \times 200$$
$$= 1\ 566\ 600$$

$$\therefore \qquad R_{BV} = 1\ 566\ 600 / 800 = 1958 \text{ N}$$

and $\qquad R_{AV} = 4333 - 1958 = 2375 \text{ N}$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $\qquad M_{CV} = R_{AV} \times 200 = 2375 \times 200$
$$= 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $\qquad M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3 \text{ N-mm}$

The bending moment diagram for vertical loading is shown in Fig. 14.6 (*e*).

Now consider the horizontal loading at *C* and *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 800 = 4000\ (800 - 250) + 849 \times 200 = 2\ 369\ 800$$

$$\therefore \qquad R_{BH} = 2\ 369\ 800 / 800 = 2963 \text{ N}$$

and $\qquad R_{AH} = 4849 - 2963 = 1886 \text{ N}$

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $\qquad M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\ 200 \text{ N-mm}$

B.M. at *D*, $\qquad M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\ 750 \text{ N-mm}$

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2}$$
$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\ 750)^2}$$
$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (*g*). We see that the bending moment is maximum at *D*, therefore

Maximum B.M., $\qquad M = M_D = 887\ 874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$$\therefore \quad d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3 \quad \text{or} \quad d = 52.4 \text{ say } 55 \text{ mm}$$ **Ans.**

Example 14.11. *A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 200 r.p.m. ; AB = 750 mm ; T_D = 30 ; m_D = 5 mm ; BD = 100 mm ; T_C = 100 ; m_C = 5 mm ; AC = 150 mm ; τ = 54 MPa = 54 N/mm^2

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

∴ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716 × 10^3 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 750 = 2870 \times 150$$

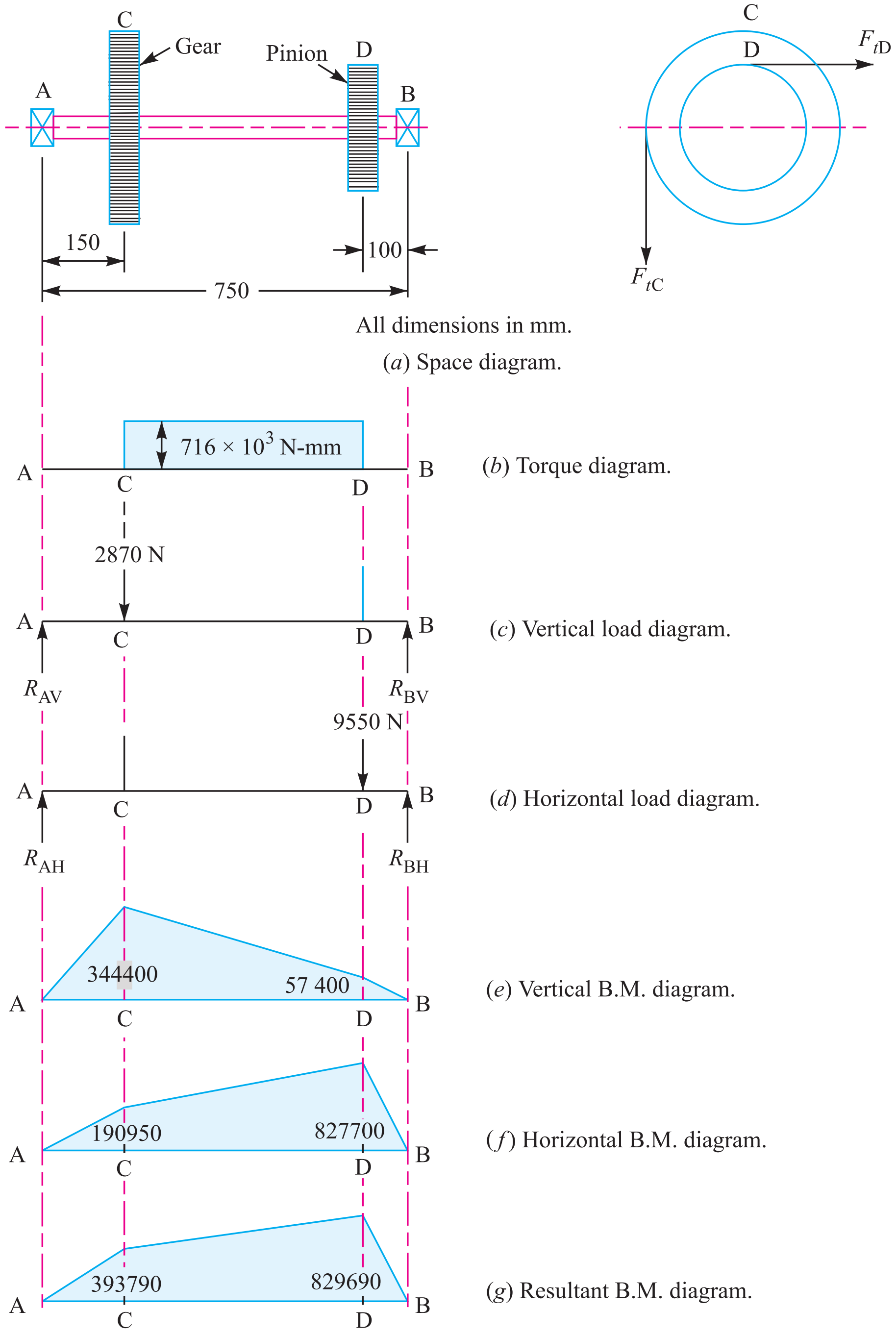


Fig. 14.8

∴ $R_{BV} = 2870 \times 150 / 750 = 574$ N

and $R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\ 400$ N-mm

B.M. at *D*, $M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\ 400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 750 = 9550\ (750 - 100) = 9550 \times 650$$

$$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277 \text{ N}$$

and $R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\ 950$ N-mm

B.M. at *D*, $M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\ 700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\ 400)^2 + (190\ 950)^2}$$
$$= 393\ 790 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\ 400)^2 + (827\ 700)^2}$$
$$= 829\ 690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 829\ 690 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\ 690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$$\therefore \quad d^3 = 1096 \times 10^3/10.6 = 103.4 \times 10^3$$

or d = 47 say 50 mm **Ans.**

14.12 Shafts Subjected to Fluctuating Loads

Crankshaft

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (*T*) and bending moment (*M*). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Table 14.2. Recommended values for K_m and K_t.

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 14.12. *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 200 r.p.m. ; W = 900 N ; L = 2.5 m ; τ = 42 MPa = 42 N/mm^2 ; σ_b = 56 MPa = 56 N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$
$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3 \text{ or } d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3 \text{ or } d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

d = 53.4 say 55 mm **Ans.**

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

K_m = 1.5 and K_t = 1

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3 \text{ or } d = 53.6 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \frac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 57.7 say 60 mm **Ans.**

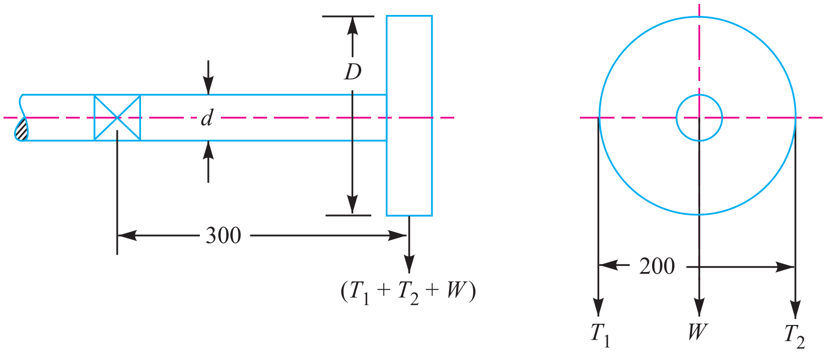
Example 14.13. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.*

Solution. Given : W = 200 N ; L = 300 mm ; D = 200 mm or R = 100 mm ; P = 1 kW = 1000 W ; N = 120 r.p.m. ; θ = 180° = π rad ; μ = 0.3 ; K_m = 1.5 ; K_t = 2 ; τ = 35 MPa = 35 N/mm^2

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Fig. 14.9

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2)\, R = (T_1 - T_2)\, 100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \quad \text{...}(ii)$$

...(Taking antilog of 0.4098)

From equations *(i)* and *(ii)*, we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm}$$ **Ans.**

Example 14.14. *Fig. 14.10 shows a shaft carrying a pulley A and a gear B and supported in two bearings C and D. The shaft transmits 20 kW at 150 r.p.m. The tangential force F_t on the gear B acts vertically upwards as shown.*

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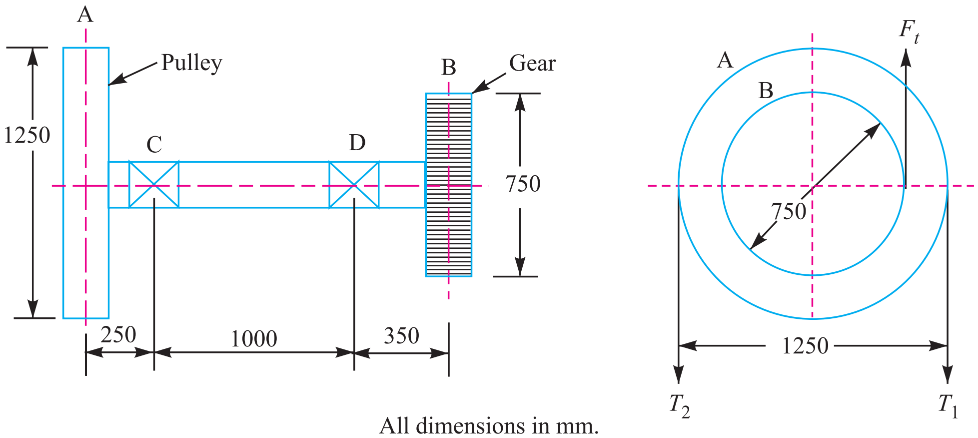
The pulley delivers the power through a belt to another pulley of equal diameter vertically below the pulley A. The ratio of tensions T_1 / T_2 is equal to 2.5. The gear and the pulley weigh 900 N and 2700 N respectively. The permissible shear stress for the material of the shaft may be taken as 63 MPa. Assuming the weight of the shaft to be negligible in comparison with the other loads, determine its diameter. Take shock and fatigue factors for bending and torsion as 2 and 1.5 respectively.

Solution. Given : P = 20 kW = 20×10^3 W ; N = 150 r.p.m. ; $T_1/T_2 = 2.5$; W_B = 900 N ; W_A = 2700 N ; τ = 63 MPa = 63 N/mm^2 ; $K_m = 2$; $K_t = 1.5$; D_B = 750 mm or R_B = 375 mm ; D_A = 1250 mm or R_A = 625 mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{20 \times 10^3 \times 60}{2 \pi \times 150} = 1273 \text{ N-m} = 1273 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Fig. 14.10

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt on pulley A.

Since the torque on the pulley is same as that of shaft (*i.e.* 1273×10^3 N-mm), therefore

$$(T_1 - T_2) R_A = 1273 \times 10^3 \quad \text{or} \quad T_1 - T_2 = 1273 \times 10^3 / 625 = 2037 \text{ N} \qquad \text{...}(i)$$

Since $T_1 / T_2 = 2.5$ or $T_1 = 2.5\, T_2$, therefore

$$2.5\, T_2 - T_2 = 2037 \quad \text{or} \quad T_2 = 2037/1.5 = 1358 \text{ N} \qquad \text{...[From equation } (i)]$$

and $T_1 = 2.5 \times 1358 = 3395$ N

∴ Total vertical load acting downward on the shaft at A

$$= T_1 + T_2 + W_A = 3395 + 1358 + 2700 = 7453 \text{ N}$$

Assuming that the torque on the gear B is same as that of the shaft, therefore the tangential force acting vertically upward on the gear B,

$$F_t = \frac{T}{R_B} = \frac{1273 \times 10^3}{375} = 3395 \text{ N}$$

Since the weight of gear B (W_B = 900 N) acts vertically downward, therefore the total vertical load acting upward on the shaft at B

$$= F_t - W_B = 3395 - 900 = 2495 \text{ N}$$

Now let us find the reactions at the bearings C and D. Let R_C and R_D be the reactions at C and D respectively. A little consideration will show that the reaction R_C will act upward while the reaction R_D act downward as shown in Fig. 14.11.

Top

Taking moments about *D*, we get

$$R_C \times 1000 = 7453 \times 1250 + 2495 \times 350 = 10.2 \times 10^6$$

$$\therefore \quad R_C = 10.2 \times 10^6 / 1000 = 10\,200 \text{ N}$$

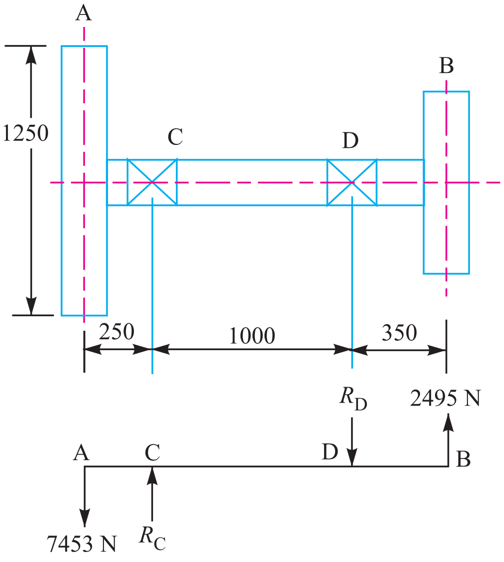


Fig. 14.11

For the equilibrium of the shaft,

$$R_D + 7453 = R_C + 2495 = 10\,200 + 2495 = 12\,695$$

$$\therefore \quad R_D = 12\,695 - 7453 = 5242 \text{ N}$$

We Know that B.M. at *A* and *B*

$$= 0$$

B.M. at *C* $= 7453 \times 250 = 1863 \times 10^3$ N-mm

B.M. at *D* $= 2495 \times 350 = 873 \times 10^3$ N-mm

We see that the bending moment is maximum at *C*.

$$\therefore \quad \text{Maximum B.M.} = M = M_C = 1863 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1863 \times 10^3)^2 + (1.5 \times 1273 \times 10^3)^2}$$

$$= 4187 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$4187 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 63 \times d^3 = 12.37\, d^3.$$

$$\therefore \quad d^3 = 4187 \times 10^3 / 12.37 = 338 \times 10^3$$

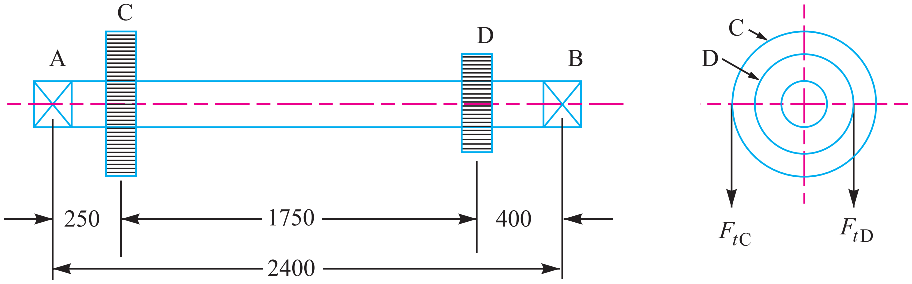
or $\quad d = 69.6$ say 70 mm **Ans.**

Example 14.15. *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft* *transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.*

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Solution. Given : $AC = 250$ mm ; $BD = 400$ mm ; $D_C = 600$ mm or $R_C = 300$ mm ; $D_D = 200$ mm or $R_D = 100$ mm ; $AB = 2400$ mm ; $P = 20$ kW $= 20 \times 10^3$ W ; $N = 120$ r.p.m ; $\sigma_t = 100$ MPa $= 100$ N/mm^2 ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $W_C = 950$ N ; $W_D = 350$ N ; $K_m = 1.5$; $K_t = 1.2$

The shaft supported in bearings and carrying gears is shown in Fig. 14.12.



All dimensions in mm.

Fig. 14.12

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears C and D is same as that of the shaft, therefore the tangential force acting at gear C,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

Car rear axle.

and total load acting downwards on the shaft at *C*

$$= F_{tC} + W_C = 5300 + 950 = 6250 \text{ N}$$

Similarly tangential force acting at gear *D*,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at *D*

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250 \text{ N}$$

Now assuming the shaft as a simply supported beam as shown in Fig. 14.13, the maximum bending moment may be obtained as discussed below :

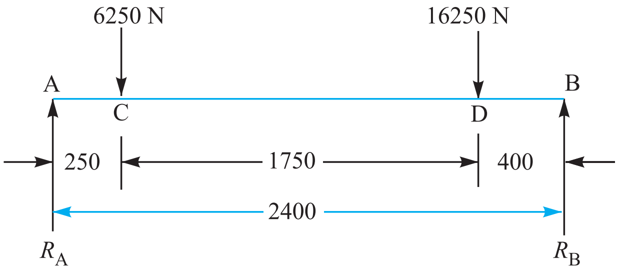


Fig. 14.13

Let R_A and R_B = Reactions at *A* and *B* respectively.

∴ $R_A + R_B$ = Total load acting downwards at *C* and *D*

$= 6250 + 16\,250 = 22\,500$ N

Now taking moments about *A*,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

∴ $R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190$ N

and $R_A = 22\,500 - 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at *C* or *D*.

We know that bending moment at *C*,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 \text{ N-mm}$$

Bending moment at *D*,

$${}^*M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

∴ Maximum bending moment transmitted by the shaft,

$$M = M_D = 5676 \times 10^3 \text{ N-mm}$$

Let *d* = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$$= 8725 \times 10^3 \text{ N-mm}$$

---

* The bending moment at *D* may also be calculated as follows :

$M_D = R_A \times 2000 - (\text{Total load at } C)\ 1750$

We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore \qquad d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\,(K_m \times M + T_e)$$

$$= \tfrac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right] = 8620 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\ d^3 \qquad \text{...(Taking } \sigma_b = \sigma_t)$$

$$\therefore \qquad d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3 \text{ or } d = 95.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 95.7 say 100 mm **Ans.**

Example 14.16. *A hoisting drum 0.5 m in diameter is keyed to a shaft which is supported in two bearings and driven through a 12 : 1 reduction ratio by an electric motor. Determine the power of the driving motor, if the maximum load of 8 kN is hoisted at a speed of 50 m/min and the efficiency of the drive is 80%. Also determine the torque on the drum shaft and the speed of the motor in r.p.m. Determine also the diameter of the shaft made of machinery steel, the working stresses of which are 115 MPa in tension and 50 MPa in shear. The drive gear whose diameter is 450 mm is mounted at the end of the shaft such that it overhangs the nearest bearing by 150 mm. The combined shock and fatigue factors for bending and torsion may be taken as 2 and 1.5 respectively.*

Solution. Given : D = 0.5 m or R = 0.25 m ; Reduction ratio = 12 : 1 ; W = 8 kN = 8000 N ; v = 50 m/min ; η = 80% = 0.8 ; σ_t = 115 MPa = 115 N/mm^2 ; τ = 50 MPa = 50 N/mm^2 ; D_1 = 450 mm or R_1 = 225 mm = 0.225 m ; Overhang = 150 mm = 0.15 m ; K_m = 2 ; K_t = 1.5

Power of the driving motor

We know that the energy supplied to the hoisting drum per minute

$$= W \times v = 8000 \times 50 = 400 \times 10^3 \text{ N-m/min}$$

$\therefore$ Power supplied to the hoisting drum

$$= \frac{400 \times 10^3}{60} = 6670 \text{ W} = 6.67 \text{ kW} \qquad \text{...}(\because 1 \text{ N-m/s} = 1 \text{ W})$$

Since the efficiency of the drive is 0.8, therefore power of the driving motor

$$= \frac{6.67}{0.8} = 8.33 \text{ kW}$$ **Ans.**

Torque on the drum shaft

We know that the torque on the drum shaft,

T = $W.R$ = 8000 × 0.25 = 2000 N-m **Ans.**

Speed of the motor

Let N = Speed of the motor in r.p.m.

We know that angular speed of the hoisting drum

$$= \frac{\text{Linear speed}}{\text{Radius of the drum}} = \frac{v}{R} = \frac{50}{0.25} = 200 \text{ rad / min}$$

Since the reduction ratio is 12 : 1, therefore the angular speed of the electric motor,

$$\omega = 200 \times 12 = 2400 \text{ rad/min}$$

and speed of the motor in r.p.m.,

$$N = \frac{\omega}{2\pi} = \frac{2400}{2\pi} = 382 \text{ r.p.m.}$$ **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

Since the torque on the drum shaft is 2000 N-m, therefore the tangential tooth load on the drive gear,

$$F_t = \frac{T}{R_1} = \frac{2000}{0.225} = 8900 \text{ N}$$

Assuming that the pressure angle of the drive gear in 20°, therefore the maximum bending load on the shaft due to tooth load

$$= \frac{F_t}{\cos 20°} = \frac{8900}{0.9397} = 9470 \text{ N}$$

Since the overhang of the shaft is 150 mm = 0.15 m, therefore bending moment at the bearing,

$$M = 9470 \times 0.15 = 1420 \text{ N-m}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1420)^2 + (1.5 \times 2000)^2} = 4130 \text{ N-m} = 4130 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$4130 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 50 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 4130 \times 10^3 / 9.82 = 420.6 \times 10^3 \quad \text{or} \quad d = 75 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}(K_m \times M + T_e)$$

$$= \frac{1}{2}(2 \times 1420 + 4130) = 3485 \text{ N-m} = 3485 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$3485 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 115 \times d^3 = 11.3\, d^3$$

$$\therefore \quad d^3 = 3485 \times 10^3 / 11.3 = 308.4 \times 10^3 \quad \text{or} \quad d = 67.5 \text{ mm}$$

Taking the larger of the two values, we have

d = 75 mm **Ans.**

Example 14.17. *A solid steel shaft is supported on two bearings 1.8 m apart and rotates at 250 r.p.m. A 20° involute gear D, 300 mm diameter is keyed to the shaft at a distance of 150 mm to the left on the right hand bearing. Two pulleys B and C are located on the shaft at distances of 600 mm and 1350 mm respectively to the right of the left hand bearing. The diameters of the pulleys B and C are 750 mm and 600 mm respectively. 30 kW is supplied to the gear, out of which 18.75 kW is taken off at the pulley C and 11.25 kW from pulley B. The drive from B is vertically downward while from C the drive is downward at an angle of 60° to the horizontal. In both cases the belt tension ratio is 2 and the angle of lap is 180°. The combined fatigue and shock factors for torsion and bending may be taken as 1.5 and 2 respectively.*

Design a suitable shaft taking working stress to be 42 MPa in shear and 84 MPa in tension.

Solution. Given : $PQ = 1.8$ m ; N = 250 r.p.m ; $\alpha_D = 20°$; $D_D = 300$ mm or $R_D = 150$ mm = 0.15 m ; $QD = 150$ mm = 0.15 m ; $PB = 600$ mm = 0.6 m ; $PC = 1350$ mm = 1.35 m ; $D_B = 750$ mm or $R_B = 375$ mm = 0.375 m ; $D_C = 600$ mm or $R_C = 300$ mm = 0.3 m ; $P_D = 30$ kW = 30×10^3 W ; $P_C = 18.75$ kW = 18.75×10^3 W ; $P_B = 11.25$ kW = 11.25×10^3 W ; $T_{B1}/T_{B2} = T_{C1}/T_{C2} = 2$; $\theta = 180° = \pi$ rad ; $K_t = 1.5$; $K_m = 2$; $\tau = 42$ MPa = 42 N/mm^2 ; $\sigma_t = 84$ MPa = 84 N/mm^2

First of all, let us find the total loads acting on the gear *D* and pulleys *C* and *B* respectively.

For gear D

We know that torque transmitted by the gear *D*,

$$T_D = \frac{P_D \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 250} = 1146 \text{ N-m}$$

∴ Tangential force acting on the gear *D*,

$$F_{tD} = \frac{T_D}{R_D} = \frac{1146}{0.15} = 7640 \text{ N}$$

and the normal load acting on the gear tooth,

$$W_D = \frac{F_{tD}}{\cos 20°} = \frac{7640}{0.9397} = 8130 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.14. Resolving the normal load vertically and horizontally, we have

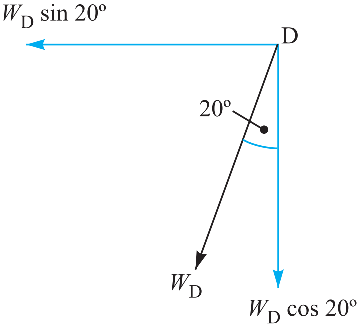


Fig. 14.14

Vertical component of W_D

$$= W_D \cos 20° = 8130 \times 0.9397 = 7640 \text{ N}$$

Horizontal component of W_D

$$= W_D \sin 20° = 8130 \times 0.342 = 2780 \text{ N}$$

For pulley C

We know that torque transmitted by pulley *C*,

$$T_C = \frac{P_C \times 60}{2\pi N} = \frac{18.75 \times 10^3 \times 60}{2\pi \times 250} = 716 \text{ N-m}$$

Let T_{C1} and T_{C2} = Tensions in the tight side and slack side of the belt for pulley*C*.

We know that torque transmitted by pulley *C* (T_C),

$$716 = (T_{C1} - T_{C2})\, R_C = (T_{C1} - T_{C2})\, 0.3$$

∴ $T_{C1} - T_{C2} = 716 / 0.3 = 2387$ N ...(*i*)

Since $T_{C1} / T_{C2} = 2$ or $T_{C1} = 2\, T_{C2}$, therefore from equation (*i*), we have

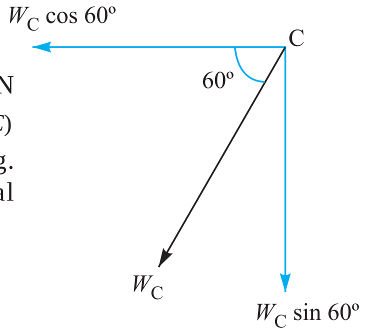
$T_{C2} = 2387$ N ; and $T_{C1} = 4774$ N

∴ Total load acting on pulley *C*,

$$W_C = T_{C1} + T_{C2} = 4774 + 2387 = 7161 \text{ N}$$

...(Neglecting weight of pulley *C*)

This load acts at 60° to the horizontal as shown in Fig. 14.15. Resolving the load W_C into vertical and horizontal components, we have



Vertical component of W_C

$$= W_C \sin 60° = 7161 \times 0.866$$
$$= 6200 \text{ N}$$

Trainwheels and Axles

and horizontal component of W_C

$$= W_C \cos 60° = 7161 \times 0.5$$

$$= 3580 \text{ N}$$

For pulley B

We know that torque transmitted by pulley *B*,

$$T_B = \frac{P_B \times 60}{2\pi N} = \frac{11.25 \times 10^3 \times 60}{2\pi \times 250} = 430 \text{ N-m}$$

Let T_{B1} and T_{B2} = Tensions in the tight side and slack side of the belt for pulley *B*.

We know that torque transmitted by pulley *B* (T_B),

$$430 = (T_{B1} - T_{B2})\, R_B = (T_{B1} - T_{B2})\, 0.375$$

$$\therefore \quad T_{B1} - T_{B2} = 430 / 0.375 = 1147 \text{ N} \quad \text{...(ii)}$$

Since $T_{B1} / T_{B2} = 2$ or $T_{B1} = 2T_{B2}$, therefore from equation (*ii*), we have

$$\text{T}_{B2} = 1147 \text{ N, and } T_{B1} = 2294 \text{ N}$$

∴ Total load acting on pulley *B*,

$$W_B = T_{B1} + T_{B2} = 2294 + 1147 = 3441 \text{ N}$$

This load acts vertically downwards.

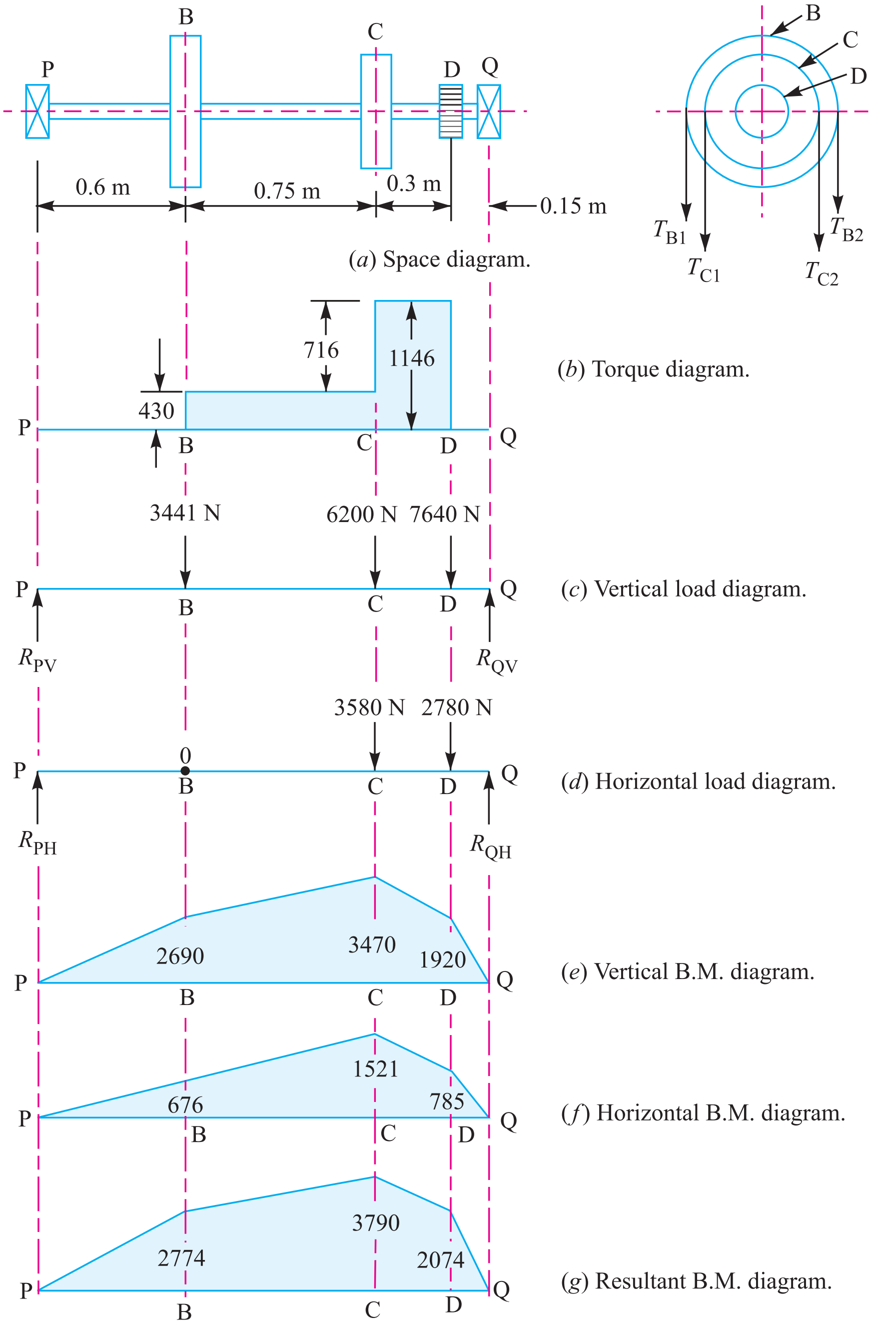
From above, we may say that the shaft is subjected to the vertical and horizontal loads as follows :

Type of loading	*Load in N*		
	At D	*At C*	*At B*
Vertical	7640	6200	3441
Horizontal	2780	3580	0

The vertical and horizontal load diagrams are shown in Fig. 14.16 (*c*) and (*d*).

First of all considering vertical loading on the shaft. Let R_{PV} and R_{QV} be the reactions at bearings *P* and *Q* respectively for vertical loading. We know that

$$R_{PV} + R_{QV} = 7640 + 6200 + 3441 = 17\ 281 \text{ N}$$



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagram.

Fig. 14.16

Taking moments about *P*, we get

$$R_{QV} \times 1.8 = 7640 \times 1.65 + 6200 \times 1.35 + 3441 \times 0.6 = 23\ 041$$

$\therefore$ $R_{QV} = 23\ 041 / 1.8 = 12\ 800$ N

and $R_{PV} = 17\ 281 - 12\ 800 = 4481$ N

We know that B.M. at *P* and *Q*,

$$M_{PV} = M_{QV} = 0$$

B.M. at *B*, $M_{BV} = 4481 \times 0.6 = 2690$ N-m

B.M. at *C*, $M_{CV} = 4481 \times 1.35 - 3441 \times 0.75 = 3470$ N-m

and B.M. at *D*, $M_{DV} = 12\ 800 \times 0.15 = 1920$ N-m

The bending moment diagram for vertical loading is shown in Fig. 14.16 (*e*).

Now considering horizontal loading. Let R_{PH} and R_{QH} be the reactions at the bearings *P* and *Q* respectively for horizontal loading. We know that

$$R_{PH} + R_{QH} = 2780 + 3580 = 6360 \text{ N}$$

Taking moments about *P*, we get

$$R_{QH} \times 1.8 = 2780 \times 1.65 + 3580 \times 1.35 = 9420 \text{ N}$$

$\therefore$ $R_{QH} = 9420 / 1.8 = 5233$ N

and $R_{PH} = 6360 - 5233 = 1127$ N

We know that B.M. at *P* and *Q*,

$$M_{PH} = M_{QH} = 0$$

B.M. at *B*, $M_{BH} = 1127 \times 0.6 = 676$ N-m

B.M. at *C*, $M_{CH} = 1127 \times 1.35 = 1521$ N-m

and B.M. at *D*, $M_{DH} = 5233 \times 0.15 = 785$ N-m

The bending moment diagram for horizontal loading is shown in Fig. 14.16 (*f*).

The resultant bending moments for the points *B*, *C* and *D* are as follows :

$$\text{Resultant B.M. at } B = \sqrt{(M_{BV})^2 + (M_{BH})^2} = \sqrt{(2690)^2 + (676)^2} = 2774 \text{ N-m}$$

$$\text{Resultant B.M. at } C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(3470)^2 + (1521)^2} = 3790 \text{ N-m}$$

$$\text{Resultant B.M. at } D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(1920)^2 + (785)^2} = 2074 \text{ N-m}$$

From above we see that the resultant bending moment is maximum at *C*.

$\therefore$ $M = M_C = 3790$ N-m

and maximum torque at *C*,

T = Torque corresponding to 30 kW = T_D = 1146 N-m

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \sqrt{(2 \times 3790)^2 + (1.5 \times 1146)^2}$$

$$= 7772 \text{ N-m} = 7772 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment (T_e),

$$7772 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 7772 \times 10^3/8.25 = 942 \times 10^3$ or $d = 98$ mm

Again, we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\,(K_m \times M + T_e)$$

$$= \tfrac{1}{2}\,(2 \times 3790 + 7772) = 7676 \text{ N-m} = 7676 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$7676 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 84 \times d^3 = 8.25\ d^3$$

$$\therefore \qquad d^3 = 7676 \times 10^3 / 8.25 = 930 \times 10^3 \text{ or } d = 97.6 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 98 \text{ say } 100 \text{ mm}$$ **Ans.**

14.13 Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M\,.\,y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\,d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... } (\because k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi\,d^3} + \frac{4\,F}{\pi\,d^2} = \frac{32}{\pi\,d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32\,M_1}{\pi\,d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32\,M_1}{\pi (d_o)^3\,(1 - k^4)}$$

$$\text{...}\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1 + k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

∴ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \, (1 - k^2)} \qquad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044 \, (L/K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

$$\text{**Column factor,} \quad \alpha = \frac{\sigma_y \, (L/K)^2}{C \pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha \, F \, d_o \, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F \, d_o \, (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F \, d_o \, (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\,\right]$$

$$= \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \, (1 - k^4)$$

It may be noted that for a solid shaft, k = 0 and $d_0 = d$. When the shaft carries no axial load, then F = 0 and when the shaft carries axial tensile load, then α = 1.

Example 14.18. *A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.*

Solution. Given : T = 1.5 kN-m = 1.5 × 10^3 N-m ; M = 3 kN-m = 3 × 10^3 N-m ; F = 10 kN = 10 × 10^3 N ; $k = d_i / d_o = 0.5$; d_o = 80 mm = 0.08 m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from Table 14.2, we find that

K_m =1.5 ; and K_t = 1.0

---

* The value of column factor (α) for tensile load is unity.

** It is an Euler's formula for long columns.

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F\, d_o\, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08\,(1 + 0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because$ α = 1, for axial tensile loading)

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(80)^3\,(1 - 0.5^4) = 94\,260\,\tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6$ MPa **Ans.**

Crankshaft inside the crank-case

Example 14.19. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. *The maximum shear stress developed in the shaft, and*
2. *The angular twist between the bearings.*

Solution. Given : d_o = 0.5 m ; d_i = 0.3 m ; P = 5600 kW = 5600 × 10^3 W ; L = 6 m ; N = 150 r.p.m. ; F = 500 kN = 500 × 10^3 N ; W = 70 kN = 70 × 10^3 N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\,460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\,500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2][(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

∴ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\ (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\ (1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\ [1 - (0.6)^4] = 0.02\,\tau$$

∴ $\tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19$ MPa **Ans.**

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \text{ } \textbf{Ans.}$$

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Example 14.20. *A hollow steel shaft is to transmit 20 kW at 300 r.p.m. The loading is such that the maximum bending moment is 1000 N-m, the maximum torsional moment is 500 N-m and axial compressive load is 15 kN. The shaft is supported on rigid bearings 1.5 m apart. The maximum permissible shear stress on the shaft is 40 MPa. The inside diameter is 0.8 times the outside diameter. The load is cyclic in nature and applied with shocks. The values for the shock factors are* $K_t = 1.5$ *and* $K_m = 1.6$.

Solution. Given : *P = 20 kW ; *N = 300 r.p.m. ; M = 1000 N-m = 1000 × 10^3 N-mm ; T = 500 N-m = 500 × 10^3 N-mm ; F = 15 kN = 15 000 N ; L = 1.5 m = 1500 mm ; τ = 40 MPa = 40 N/mm^2 ; $d_i = 0.8\ d_o$ or $k = d_i/d_o = 0.8$; $K_t = 1.5$; $K_m = 1.6$

Let d_o = Outside diameter of the shaft, and

d_i = Inside diameter of the shaft = 0.8 d_o ...(Given)

We know that moment of inertia of a hollow shaft,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]$$

and cross-sectional area of the hollow shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]$$

∴ Radius of gyration of the hollow shaft,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}[(d_o)^4 - (d_i)^4]}{\frac{\pi}{4}[(d_o)^2 - (d_i)^2]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}} = \sqrt{\frac{(d_o)^2 + (d_i)^2}{16}}$$

$$= \frac{d_o}{4}\sqrt{1 + \left(\frac{d_i}{d_o}\right)^2} = \frac{d_o}{4}\sqrt{1 + (0.8)^2} = 0.32\ d_o$$

and column factor for compressive loads,

$$\alpha = \frac{1}{1 - 0.0044\,(L/K)} = \frac{1}{1 - 0.0044\,(1500/0.32\,d_o)}$$

$$= \frac{1}{1 - 20.6/d_o} = \frac{d_o}{d_o - 20.6}$$

We know that equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F\,d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.6 \times 1000 \times 10^3 + \frac{\left(\frac{d_o}{d_o - 20.6}\right) 15000 \times d_o\,(1 + 0.8^2)}{8}\right]^2 + (1.5 \times 500 \times 10^3)^2}$$

$$= \sqrt{\left[1600 \times 10^3 + \frac{3075\,(d_o)^2}{d_o - 20.6}\right]^2 + (750 \times 10^3)^2} \qquad \text{...}(i)$$

---

* Superfluous data.

Top

We also know that equivalent twisting moment for a hollow shaft,

$$T_e = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 40\, (d_o)^3\, (1 - 0.8^4) = 4.65\, (d_o)^3 \qquad \ldots(ii)$$

Equating equations ***(i)*** and ***(ii)***, we have

$$4.65\, (d_o)^3 = \sqrt{\left[1600 \times 10^3 + \frac{3075\, (d_o)^2}{d_o - 20.6}\right]^2 + (750 \times 10^3)^2} \qquad \ldots(iii)$$

Solving this expression by hit and trial method, we find that

$$d_o = 76.32 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

and $d_i = 0.8\, d_o = 0.8 \times 80 = 64$ mm **Ans.**

Note : In order to find the minimum value of d_o to be used for the hit and trial method, determine the equivalent twisting moment without considering the axial compressive load. We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \sqrt{(1.6 \times 1000 \times 10^3)^2 + (1.5 \times 500 \times 10^3)^2} \quad \ldots(iv)$$

$$= 1767 \times 10^3 \text{ N-mm}$$

Equating equations ***(ii)*** and ***(iv)***,

$$4.65 (d_o)^3 = 1767 \times 10^3 \quad \text{or} \quad (d_o)^3 = 1767 \times 10^3 / 4.65 = 380 \times 10^3$$

$$\therefore \quad d_o = 72.4 \text{ mm}$$

Thus the value of d_o to be substituted in equation (*iii*) must be greater than 72.4 mm.

14.14 Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G\,.\,\theta}{L} \quad \text{or} \quad \theta = \frac{T\,.\,L}{J\,.\,G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad \ldots\text{(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad \ldots\text{(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity*. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then

Air acclerating downwards, pushed by the rotating blades, produced an upwards reaction that lifts the helicopter.

Note : This picture is given as additional information and is not a direct example of the current chapter.

the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m^2 $= 84 \times 10^3$ N/mm^2

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

∴ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\ (35)^3 = 8420\ \tau$$

∴ $\tau = 47\ 740 / 8420 = 5.67$ N/mm^2 = 5.67 MPa **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1 / 2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \qquad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \qquad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation **(*i*)** by equation **(*ii*)**, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \qquad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(iv)$$

Dividing equation **(*iii*)** by equation **(*iv*)**, we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

The propeller shaft of this heavy duty helicopter is subjected to very high torsion.

27/02/2020

UNIT - 5

COUPLINGS

coupling is a device For connecting the ends of two shafts together

Types of couplings: couplings are two types.

1, Rigid coupling
2, Flexible coupling.

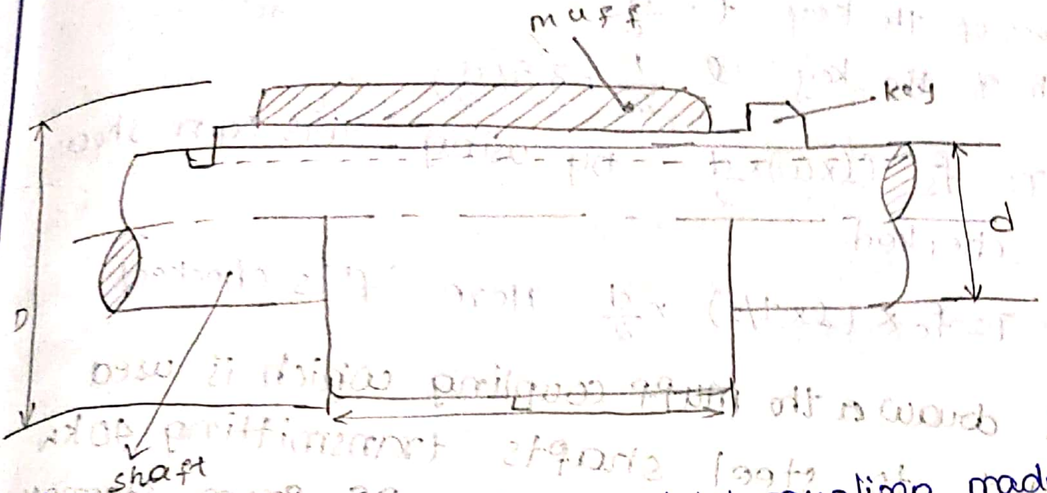
Rigid coupling: It is used to connect two shafts which are perfectly alingned i.e collinear. These couplings donot permit any mis alignment of shafts that common forms of rigid coupling are

1, Muff coupling.
2, split muff (or) clamp (or) compression coupling.
3, Flange coupling.

Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment i.e axis are not colinear. These couplings permit mis alignment and posses flexibility The common forms of flexible couplings are.

i, Hold ham couplings (permits small lateral mis alignment).
ii, universal couplings (permits small angular miss alignment).
iii, Bushed pin type coupling (absorbed shafts and permits small amount of angular and lateral mis alignment).

Design of muff coupling (sleeve coupling):



27/10/2020

It is the simplest type of rigid coupling made of cast iron it consists of a hallow cylinder whose inner diameter is the same as that of the shaft it is fitted over the ends of the two shafts by means of a "Gib head key."

Design of shaft: power transmitted by the shaft is $P = \frac{2\pi N T}{60}$ by using this eq.n torque 'T' is calculated

Torque $T = \cancel{\frac{\pi\, d^3}{16\, d^3 \cdot F_s}}$ $T = \frac{\pi}{16 \cdot d^3 \cdot F_s}$

Heare d is calculated i.e is diameter of the shaft.

Design of sleeve (or) muff: The muff is consider hallow shaft. $D = 2d + 13$ & $L = 3.5\, d$

D = diameter of the hub

L = length of the hub. $\because F_s$ = shear stress.

Torque, $T = \frac{\pi}{16} \frac{(D^4 - d^4)}{d} \cdot F_s$

By using the above eq.n shear stress of hub is checked

[f= stress]

f_c = crushingstress

Design of key:

width of the key $\omega = \frac{d}{64}$

Thickness of the key $t = \frac{d}{6}$

length of the key $l = \frac{L}{2} = 3.5(\frac{d}{2})$

Torque $T = f_s \cdot (l \times \omega) \times \frac{d}{2}$ by using this eq·n shear stress is checked.

Torque $T = f_c \times (l \times t/2) \times \frac{d}{2}$ Here 'f' is checked

1) Design and draw ~~a~~ the muff coupling which is used to connected to steel shafts transmitting 40kw at 350rpm. Design shaft and muff from strength point of view and other dimensions by emphirical forumales shear stress for muff and shaft 15N/mm² and 30N/mm². Assume maximum torque to be 25% more than mean torque.

sol N = 350rpm

P = 40kw

f_s muff = 15 N/mm²

f_s shaft = 30N/mm²

$T_{max} = 1.25 \times T_{mean}$

(1) Design of shaft:

$P = \frac{2\pi NT}{60}$

$40 \times 10^3 = \frac{2\pi \times 350 \times T}{60}$

$T = 1091.84$ N-mt

$T_{max} = 1.25 \times 1091.34$

$= 1364.18$ N-m

$T = \frac{\pi}{16} \times d^3 \times f_s$

$1364.18 \times 10^3 = \frac{\pi}{16} \times d^3 \times 30$

d = 61.41 mm

=62mm

(2) Design of muff:

D = 2d + 13

= 2×62 + 13 = 137

L = 3.5×62 = 217 mm

$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$

$1364.18 \times 10^3 = \frac{\pi}{16} \times \frac{1374.62 \times f_s}{137}$

f_s = 2.8 N/mm

So, that ~~ta~~ calculated of f_s is less than given value the muff is safe.

3 design of key

$w = d/4 = 15.5$

$t = d/6 = 10.3$

$l = 3.5(d/2) = 108.5$

3 Design a Muff coupling to connect two shafts transmitting 100kw at 200rpm the permissible shearing and crushing stresses for the shaft and key materials are 50 N/mm² & 100N/mm² respectively. The material of muff is castiron with a permissible shear stress of 15N/mm². Assume that maximum torque transmitted is Equal to mean torque.

Sol $N = 200\,rpm$

$P = 100\,kw$

~~fs shaft muff = 50N/mm²~~

~~fs shaft = 100N/mm²~~

f_s shaft $= 50\,N/mm^2$

f_c key $= 100\,N/mm^2$

f_s shaft $= 15\,N/mm^2$

1, Design of shaft

$$P = \frac{2\pi NT}{60}$$

$$100\times10^3 = \frac{2\pi\times200\times T}{60}$$

$$T = 4774.64\ N\text{-}mt$$

$$= \frac{\pi}{16}\times d^3\times f_s$$

$$4774.64 = \frac{\pi}{16}\times d^3\times 50$$

$$d^3 = 486.34$$

$$d = 78.6\ mm$$

$$\boxed{d \cong 80\ mm}$$

2, Design of muff

$$D = 2d+13$$

$$= 2(80)+13 = 173\,mm$$

$$L = 3.5\times d$$

$$= 3.5\times80 = 280\,mm$$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$4774.64 = \frac{\pi}{16}\times\frac{(173)^4-(80)^4}{173}\times f_s$$

$$f_s = 4.92\ N/mm^2$$

3, Design of key:

$$\boxed{f_c = 2.f_s}$$

$$w = d/4$$

$$= 80/4 = 20mm$$

$$t = 20mm$$

$$l = \frac{3.5d}{2}$$

$$= 140$$

$$T = f_s\times l\times w\times\frac{d}{2}$$

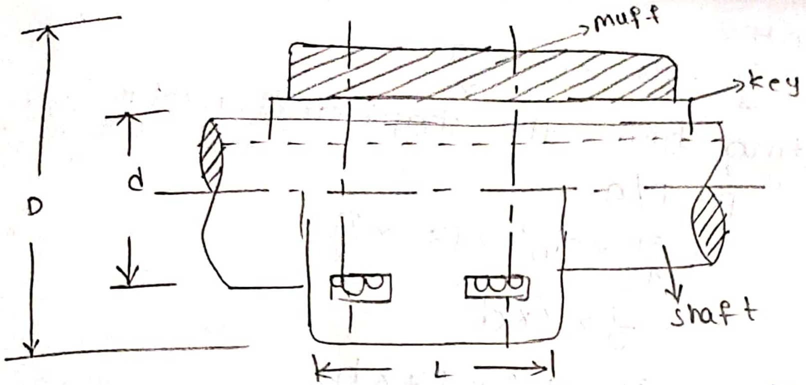
$$4774.64\times10^3 = f_s\times140\times20\times\frac{80}{2}$$

$$\boxed{f_s = 42.6}$$

$$4774.64\times10^3 = f_c\times140\times\frac{20}{2}\times\frac{80}{2}$$

$$\boxed{f_c = 85.2\ N/mm^2}$$

clamp or compression coupling: [split muff coupling]:



It is also known as split muff coupling in this case the muff (or) sleeve is made into two equal parts. The two equal parts are bolted together. The two equal parts muff made of cast iron and the two parts are joined by means of a bolts. which are made with mild steel the no.of bolts may be two, four (or) six.

29/01/2020

1, Design of shaft:

power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

By using this Eq.n Torque 'T' is calculated $T = \frac{\pi}{16 d^3 \cdot fs}$

Here 'd' is calculated i.e dia of shaft.

2, Design of sleeve or) muff:

The muff is consider hallow shaft $D = 2d + 13$ & $l = 3.5d$

D = dia of hub

l = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} fs$$

By using above eq shear stress of hub is checked.

3, Design of key:

$(w) = \frac{d}{4}$; $(t) = \frac{d}{6}$; $(L) = 35\frac{d}{2}$

$$\text{Torque } (T) = f_s (L \times w) \times \frac{d}{2}$$

By using this Eq.n shear stress is checked

$$\text{Torque } (T) = fc \times (L \times \frac{t}{2}) \times \frac{d}{2}$$

(4) Design of Bolt:

Force acting on the bolts.

$$F = \frac{\pi}{4} \times d_b^2 \times f_t$$

On each bolt

$$F = \frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}$$

Force acting b/w the shaft and muff

$$P = F/A$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d}$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times h}{4 \times L \times d}$$

Frictional force:

$\mu = \frac{F}{R_n} =$

$F = \mu \times W$

$\boxed{F = \mu \times P \times}$

$$F_F = \mu . P . A$$

$$= \mu . \frac{\pi . d_b^2 f_t . h}{4 \times L \times d} \times \frac{1}{2} \pi d . L$$

$$= \mu . \frac{\pi^2 d_b^2 \times f_t . n}{8}$$

Torque

$$T = F_F \times \perp^{ar} \text{distance}$$

$$= \frac{\mu . \pi^2 . d_b^2 . f_t \times h}{8} \times \frac{d}{2}$$

$$\boxed{T = \frac{\mu \times \pi^2 \times d_b^2 \times f_t \times h \times d}{16}}$$

1) Design a clamp coupling to transmit 30kW power at 100 rp the shaft and key The allowable shear stress for the shaft and key is 40 M.Pa and the no of Bolts connecting to the two halves are 6. The permissiable tensile stress for the bolts is 70 MPa The sufficient of friction b/w the muff & shaft surface may be taken as 0.3

$P = 30 kW$

$N = 100 rpm$

f_s shaft $= 40 M.Pa$

f_s key $= 40 M.Pa$

$n = 6$ $\quad f_t = 70 MPa$

$\mu = 0.3$

i, <u>Design of shaft:</u>

$$P = \frac{2\pi N T}{60}$$

$$30 \times 10^3 = \frac{2\pi \times 100 \times T}{60}$$

$$T = 2864.7 \text{ N-m}$$

$$T_{max} = 1.25 \times 2864.7$$

$$T = \frac{\pi}{16} \times d^3 \times f_s = 3580.9$$

$$2864.7 \times 10^3 = \frac{\pi}{16} \times d^3 \times 40.$$

$$\boxed{d = 71.6}$$

ii, $D = 2d + 13$ $\qquad C = 3.5 \times d$

$= 2(71.3) + 13$ $\qquad = 3.5 \times (71.3)$

$= 155.6$ $\qquad = 249.55$

$$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$$

$$2864.7 \times 10^3 = \frac{\pi}{16} \times \frac{155.6^4 - 71.6^4}{155.6} \times f_s$$

$$\boxed{f_s = 4.05}$$

iii, $\omega = \frac{71.6}{4}$, $t = \frac{71.6}{6}$; $C = 3.5\left(\frac{71.6}{2}\right)$

$= 17.9$ $\qquad = 11.9$ $\qquad = 125.3$

$$T = f_s . (L \times \omega) \times \frac{d}{2}$$

$$2864.7 \times 10^3 = f_s (125.3 \times 17.9) \times \frac{71.6}{2}$$

$$\boxed{f_s = 35.6}$$

$$T = f_c \left(L \times \frac{t}{2} \right) \times \frac{d}{2}$$

$$2864.7 \times 10^3 = f_c \left(125.3 \times \frac{11.9}{2}\right) \times \frac{71.6}{2}$$

$$\boxed{f_c = 107.3}$$

iv, $T = \frac{\pi^2}{16} . \mu . d_b^2 \times f_t \times n \times d$

$$2864.7 \times 10^3 = \frac{\pi^2}{16} \times 0.3 \times d_b^2 \times 70 \times 6 \times 71.6$$

$$\boxed{d_b = 22.6}$$

Design a compression coupling for shaft to transmit 1300 N-m the allowable shear stress for the shaft and key 40 mPa and the no.of bolts connecting the two parts are the permissible tensile stress for the bolt material is 70 MPa. The coefficient friction is taken as 0.3

Sol. $\mu = 0.3$

$T = 1300$ N-m

f_s shaft, key $= 40 M.Pa$

f_t bolt $= 70 M.Pa$

$n = 4$

<u>Shaft</u>

$$T = \frac{\pi}{16} \times d^3 \times f_s$$

$$1300 \times 10^3 = \frac{\pi}{16} \times d^3 \times 40$$

ii, <u>Design of muff</u>

$D = 2d + 13$ $\qquad L = 3.5d$

$= 123$ $\qquad = 3.5 \times 55$

$= 192.5 = 193$

$$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$$

$$1300 \times 10^3 = \frac{\pi}{16} \times \frac{123^4 - 55^4}{123} \times f_s$$

iii, Design of key:

$w = \frac{d}{4} =$ $l = 3.5\left(\frac{d}{2}\right)$ $t = \frac{d}{6}$

$= 13.75$ $= 96.25$ $= 9.1$

$T = fs \times (L \times w) \times \frac{d}{2}$

$1300 \times 10^3 = fs(96.25 \times 13.75) \times \frac{55}{2}$

$\boxed{fs = 35.7}$

$T = fc\left(L \times \frac{t}{2}\right) \times \frac{d}{2}$

$1300 \times 10^3 = fc\left(96.25 \times \frac{9.1}{2}\right)\left(\frac{55}{2}\right)$

$\boxed{fc = 108}$

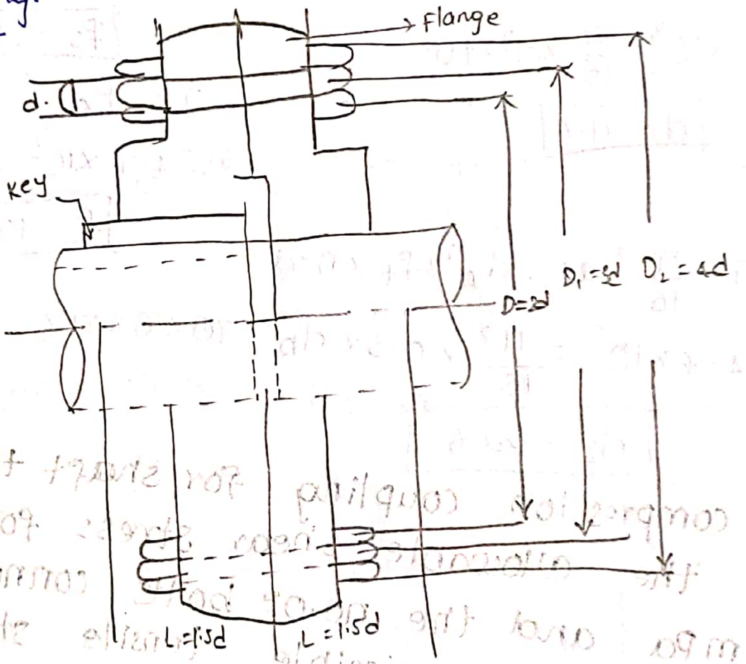
iv, Design of key:

$T = \frac{\pi^2}{16} . \mu . d_b^2 \sqrt{t} . n.d$

$1300 \times 10^3 = \frac{\pi^2}{16}(0.3) d_b^2 . 70.4$

$\boxed{d_s = 21.3}$

Flange coupling:



d = diameter of the shaft

D = Diameter of the muff

D = 2d

D_1 = pitch circle diameter of the flank

$D_1 = 3d$

D_2 = Diameter of the flange.

$D_2 = 4d$

T_f = Thickness of the flange

$T_f = 0.5d$

L = length of the flange

n = No. of the bolts

$n = 3$ for diameter upto 40mm

$n = 4$ for diameter upto 100mm

$n = 6$ for diameter upto 180mm

1. Design of shaft:

Power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

By using this Eq.n Torque 'T' calculated

$$T = \frac{\pi}{16 \cdot d^3 \cdot f_s}$$

Here 'd' is calculated is dia of shaft.

2. Design of muff:

The muff is consider hallow shaft

$D = 2d$ & $L = 1.5d$

D = dia of hub

L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

By using above Eq.n shear stress of hub is checked.

3. Design of Key: (rectangle = d/6)

$\omega = \frac{d}{4}$; $t = \frac{d}{6}$; $l = L$

Torque (T) = $f_s (l \times \omega) \times \frac{d}{2}$

By using the Eq.n shear stress is checked

Torque(T) = $f_c \times (l \times \frac{t}{2}) \times \frac{d}{2}$

By using this Eq.n crushing stress is calculated

4. Design of ~~Key~~ Flange:

Torque transmitted by the flange = circumference of the hub × thickness of the flange × Shear stress of the flange × radius of the hub.

$$T = \pi D \times t_f \times f_s \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times t_f \times f_s}$$

By using the above Eq.n Shear stress is calculated.

The thickness of the protective circumferential of flange

$T_b = 0.25d$

Design for bolts: The total load of the bolt $= \frac{\pi}{4} d_b^2 . f_{sb}$

Torque transmitted $T = \frac{\pi}{4} d_b^2 \cdot f_{sbolt} \times n \times \frac{D_1}{2}$

we can calculate diameter of bolt.

The crushing strength of the bolt $= n \times d_b \times t_f \times f_{cb}$

Torque transmitted $T = n \times t_f \times d_b \times f_{cb} \times \frac{D_1}{2}$

Design a cast iron protective type flange coupling to transmit 15KW at 900rpm from an electric motor two a compressor the service factor may be assume has 1.35 the following permissible stresses may be used. shear stress for the bolt shaft and key equal to 40 M.Pa crushing stress for bolt and key 80 M.Pa shear stress for cast iron 8M.Pa. Draw a neat sketch of the coupling.

Sol: $P = 15KW$

$N = 900rpm$

$S.F = 1.35$

f_s bolt, shaft, key = 40M.Pa

f_c bolt & key = 80M.Pa

f_s muff & flange = 8MPa

1 * **shaft**

$$P = \frac{2\pi NT}{60}$$

$$15\times10^3 = \frac{2\pi \times 900 \times T}{60}$$

$$T = \underline{159.15} N\text{-}m$$

$$T_{max} = S.F \times T$$

$$T_{max} = 1.35 \times 159.15$$

$$= 214.85 \text{ N-mt}$$

$$T = \frac{\pi}{16} \times d^3 \times f_s$$

$$d = 30.13 \text{ mm} \cong 31$$

$$\boxed{n = 3}$$

2 * **Muff:**

$D = 2d = 2(31) = 62$

$L = 1.5d = 46.5 \cong 47$

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$$

$$159.15\times10^3 = \frac{\pi}{16} \frac{62^4 - 31^4}{62} \times f_s$$

$$f_s = 4.89 \ N/mm^2$$

the design of muff is safe.

* **Key:**

$f_c = 2 \times f_s$

$f_s = 2 \times 4.89 =$

so hear taken sq.key

$w = t = \frac{d}{4} = 7.75 \cong 8$

$l = L = 1.5d = 46.5 \cong 47$

$$T = f_s \times (l \times w) \times \frac{d}{2}$$

$$f_s = \underline{36.85} \ N/mm^2$$

$$T = f_c \times (l \times \frac{t}{2}) \times \frac{d}{2}$$

f_c [illegible] N/mm^2

i) flange:

$D_1 = 3d =$

$t_f = 0.5d =$

$t_p = 0.25d$

$T = \pi D \times t_f \times f_s \times D/2$

$f_s = 2.22$ N/mm²

4 Bolt

$T = \frac{\pi}{4} \times d_b^2 \times n \times f_s \times \frac{D_1}{2}$

$d_b =$

$T = n \cdot t_f \times d_b \times f_c \times \frac{D}{2}$

Diagram

ii. Design a cast iron flang coupling to connect two shafts in order to transmit 7.5 kW at 720 rpm the following permissible stress may be assumed permissible shear stress for shaft bolt and key equal to 33 N/mm² permissible crushing stress for bolt and key 60 N/mm². The permissible shear stress for cast iron 15 N/mm².

$P = 7.5$ kW

$N = 720$ rpm

f_s shaft, bolt key $= 33$ N/mm²

f_c bolt key $= 60$ N/mm²

f_s muff, flang $= 15$ N/mm²

Shaft:

$$P = \frac{2\pi NT}{60}$$

$$\frac{7.5 \times 10^3 \times 60}{2\pi \times 720} = T$$

$T = 99.4$ N-m

$T_{max} = SF \times T$

$$T = \frac{\pi}{16} \times d^3 \times F_s$$

$$d^3 = \frac{T \times 16}{\pi \times F_s}$$

$$d^3 = \frac{99.4 \times 10^3 \times 16}{\pi \times 33}$$

$d = 24.8\text{mm} \simeq 25$

$\boxed{n:3}$

Muff

$D = 2d = 2(25) = 50$

$l = 1.5d = 1.5(25) = 37.5 \simeq 38$

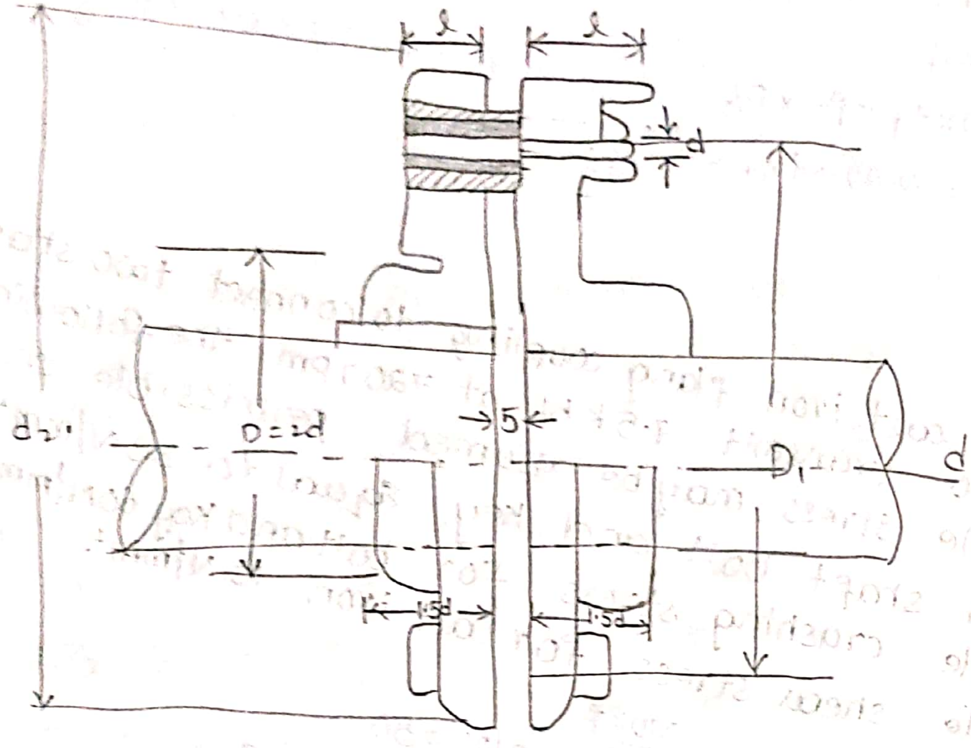
$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$$

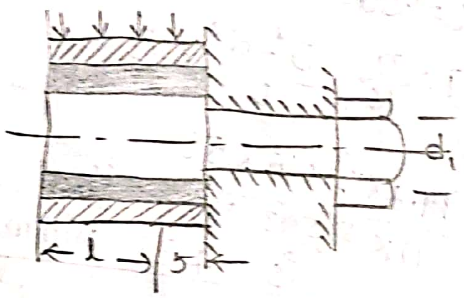
$$\frac{99.4 \times 10^3 \times 16 \times 50}{\pi \times 5559375} = f_s$$

$\boxed{f_s = 4.31 \text{ N/mm}^2}$

Flexible flange coupling:

Bushed pin flexible coupling:





A Bushed pin flexible coupling is a modification of rigid type flange coupling. the bolts of a coupling are known as pins. The rubber or leather bushes are used over the pins. The two parts of the coupling are disimillar in construction. A clearance of 5mm is left b/w the two parts of the couplings There is no rigid connection b/w them and the drive takes place through the medium of compressible rubber or leather bushes.

l = Length of the bush in the flange.
d_2 = diameter of the bush
P_b = bearing pressure on the bush or pin

D_1 = pitch circle diameter of the pins.

d_1 = diameter of pin $d_1 = \frac{0.5d}{\sqrt{n}}$

over all diameter of rubber bush

$$d_2 = d_1 + 4 + 2\times2 + 2\times6 = d_1 + 20$$

Diameter of pitch circle of the pins

$$D_1 = 2\times d + d_2 + 2\times6$$

Bearing load acting on each pin

$$W = P_b \times d_2 \times l \quad = 32l.$$

Total Bearing load on the bush or pin.

$$= W \times n$$

$$= P_b \times d_2 \times l \times n$$

Maximum Torque" transmitted by the coupling

$$T = W \times n \times \frac{D_1}{2}$$

Direct shear stress give to pure torsion in the coupling bolts

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2}$$

The bush portion of the pin act as a cantilever beam of length 'l'. assume a uniform distributed load 'W' along the bush the bending moment

$$M = W\left(\frac{l}{2} + 5\right)$$

The bending stress = $\sigma_1 = \frac{M}{Z}$

$$Z = \frac{\pi}{32} d_1^3$$

since, the pin is subjected to bending stress and shear stress. Therefore, the design must be checked either for maximum principle stress or maximum shear stress.

$$\text{Max. principle stress} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$\text{Max. shear stress} = \frac{1}{2}\left[\sqrt{\sigma_1^2 + 4\tau^2}\right].$$

The value of Maximum principle stresses varies fro

28 - 42 M.Pa

1) **Design of hub(or) muff :**

The muff is considered hallowshaft

$D = 2d$ & $L = 1.5d$

D = diameter of hub

L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

By using above Eq.n shear stress of hub is checked

2) **Design of key:**

$\omega = \frac{d}{4}$; $t = \frac{d}{6}$; $l = L$

$$\text{Torque}(T) = f_s (l \times w) \times \frac{d}{2}$$

By using the abore Eq.n shear stress is calculated

$$\text{Torque}(T) = f_c \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

By using this Eq.n crushing stress is calculated.

3) **Design of flange:**

Torque transmitted by the flange = circumference of hub × thickness of flange × shear stress of flange × radius of hub

$$T = \pi D \times t_f \times f_s \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times t_f \times f_s}$$

By using above eq.n shear stress is calaulated.

The thickness of protective circumferential flange

$$\boxed{T_b = 0.25d}$$

1, Design a bush pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32kW at 960 rpm the overall torque is 20% more than the mean torque. The material properties are as follows.

i, The allowable shear stress and crushing stress for shaft and key material is 40 M.Pa & 80 M.Pa

ii, The allowable shear stress for cast iron is 15 M.Pa

iii, The allowable bearing pressure for rubber bush 0.8 N/mm

The material for the pin is same as shaft and Key. Draw a neat sketch of coupling.

Sol. Given,

P=32KW

N=960

Tmax = 1.20 Tmean

f_s key = 40M.Pa

f_c key = 80 M.Pa

P_b = 0.8 N/mm²

f muff & flange = 15 M.Pa

Design of pin:

$$P = \frac{2\pi N T}{60}$$

$$32\times10^3 = \frac{2\pi\times960\times T}{60}$$

$$\boxed{T = 318.30}$$

Tmax = 1.20 × Tmean

Tmax = 1.20 ×318.30

$$\boxed{T_{max} = 381.96\ N\text{-}m}$$

shaft

$$T_{max} = \frac{\pi}{16}\times d^3\times f_s$$

$$381.96\times10^3 = \frac{\pi}{16}\times d^3\times 40$$

$$\boxed{d^3 = 36.5} \cong \cancel{8}\ 40$$

$$d_1 = \frac{0.5\times d}{\sqrt{n}} \qquad n=6$$

$$d_1 = \frac{0.5\times 40}{\sqrt{6}}$$

$$\boxed{d_1 = 8.1} \cong 20\ mm$$

$$d_2 = d_1 + 4 + 2\times2 + 2\times6$$

$$d_2 = 20 + 4 + 2\times2 + 2\times6$$

$$\boxed{d_2 = 40}\ mm$$

$$D_1 = 2\times d + d_2 + 2\times6$$

$$D_1 = 2\times40 + 40 + 2\times6$$

$$\boxed{D_1 = 132mm}$$

$$W = P_b \times d_2 \times l$$

$$W = 0.8 \times 40 \times l$$

$$\boxed{W = 32\,l}$$

$$T_{max} = W \times n \times \frac{D_1}{2}$$

$$381.96\times10^3 = 32\,l \times 6 \times \frac{132}{2}$$

$$l = 30.14\ mm \cong 32$$

$$W = 32 \times 30.14$$

$$\boxed{W = 964.48}\ N$$

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2}$$

$$\tau = \frac{964.48}{\frac{\pi}{4}(\overset{20}{\cancel{8.1}})^2} = 3.07$$

$$M = W\left(\frac{l}{2} + 5\right)$$

$$= 964.48\left(\frac{\overset{32}{\cancel{30.14}}}{2} + 5\right)$$

$$= 20254.08\ N\text{-}mm$$

$$\sigma = \frac{M}{z} \qquad z = \frac{\pi}{32}d_1^3$$

$$z = \frac{\pi\ (20^3)}{32\times(20)^3} = 1.227\times10^{-5}$$

$$\sigma = \frac{20254.08}{\underset{785.3}{\cancel{1.227\times10^{-5}}}} = 25.79$$

$$\text{Max. principle stress} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[25.79 + \sqrt{(25.79)^2 + 4(3.07)}\right]$$

$$= 25.9 \cong 30$$

Max. shear stress =

$$\frac{1}{2}\left[\sigma^2 + 4\tau^2\right]$$

$$= \frac{1}{2}\left[(25.79)^2 + 4(3.07)^2\right]$$

$$= 351.41$$

ii) Design of hub:-

$D = 2d$

$D = 2(40) = 80$

$l = 1.5d$

$l = (1.5)(40)$

$= 60$

$$T = \frac{\pi}{\cancel{16}}\,\frac{D^4 - d^4}{D}\cdot f_s$$

$$= \frac{\pi}{4}\,\frac{80^4 - 40^4}{\cancel{80}\ 20}\cdot\cancel{40}$$

$= 15079644.74$

Design of key:

$$\omega = \frac{d}{4} = \frac{40}{4} = 10$$

$$T = f_s \times (l \times \omega) \times \frac{d}{2}$$

$$= 40 \times (32 \times 964.48) \times 20$$

$$= 24690688$$

$$\tau = f_c \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

$= 80 \times (32 \times$

27/02/2020

UNIT - 5

COUPLINGS

coupling is a device for connecting the ends of two shafts together

Types of couplings: couplings are two types.

1, Rigid coupling
2, Flexible coupling.

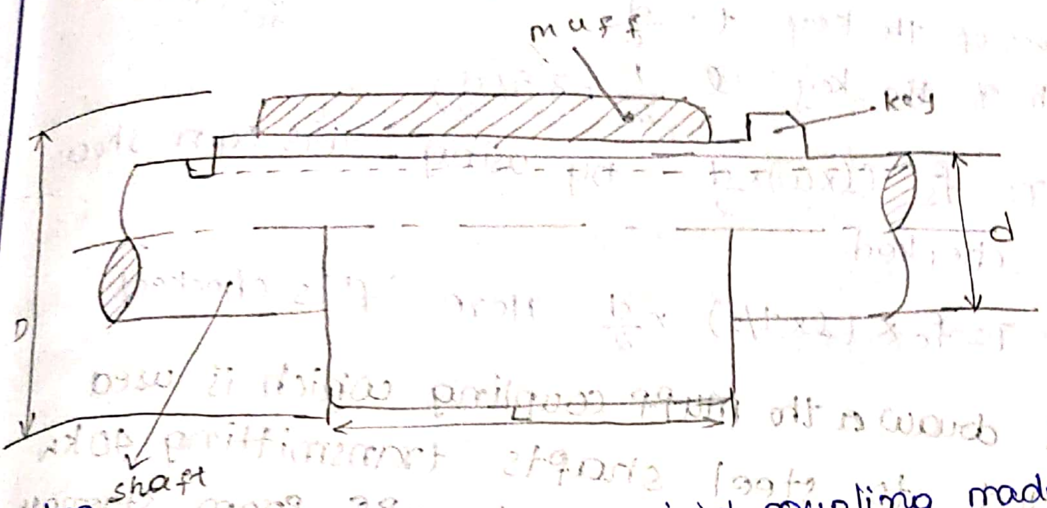
Rigid coupling: It is used to connect two shafts which are perfectly alingned i.e collinear. These couplings donot permit any mis alignment of shafts that common forms of rigid coupling are

1, Muff coupling.
2, split muff (or) clamp (or) compression coupling.
3, Flange coupling.

Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment i.e axis are not colinear. These couplings permit mis alignment and posses flexibility The common forms of flexible couplings are.

i, Hold ham couplings (permits small lateral mis alignment).
ii, universal couplings (permits small angular miss alignment).
iii, Bushed pin type coupling (absorbed shafts and permits small amount of angular and lateral mis alignment).

Design of muff coupling (sleeve coupling):



27/10/2020 It is the simplest type of rigid coupling made of cast iron it consists of a hallow cylinder whose inner diameter is the same as that of the shaft it is fitted over the ends of the two shafts by means of a "Gib head key."

Design of shaft: power transmitted by the shaft torque 'T' is

$P = \frac{2\pi N T}{60}$ by using this eq.n

calculated

Torque $T = \cancel{\frac{\pi\, d^3}{16\, d^3\, F_s}}$ $T = \frac{\pi}{16 . d^3 . F_s}$

Heare d is calculated i.e is diameter of the shaft.

i.

Design of sleeve (or) muff: The muff is consider hallow shaft. $D = 2d + 13$ & $L = 3.5\, d$

D = diameter of the hub

L = length of the hub.

$\therefore f_s$ = shear stress.

Torque, $T = \frac{\pi}{16} \frac{(D^4 - d^4)}{d} . f_s$.

By using the above eq.n shear stress of hub is checked

f = stress

Design of key:

width of the key $w = \frac{d}{64}$

Thickness of the key $t = \frac{d}{6}$

length of the key $l = \frac{L}{2} = 3.5\left(\frac{d}{2}\right)$

f_c = crushing stress

Torque $T = f_s \cdot (l \times w) \times \frac{d}{2}$ by using this eq.n shear stress is checked.

Torque $T = f_c \times (l \times t/2) \times \frac{d}{2}$ Here 'f' is checked

1) Design and draw a the muff coupling which is used to connected to steel shafts transmitting 40kw at 350rpm. Design shaft and muff from strength point of view and other dimensions by emphexical formulaes shear stress for muff and shaft 150N/mm² and 30N/mm². Assume maximum torque to be 25% more than mean torque.

sol N = 350rpm

P = 40kw

f_s muff = 15 N/mm²

f_s shaft = 30N/mm²

$T_{max} = 1.25 \times T_{mean}$

1) Design of shaft:

$$P = \frac{2\pi NT}{60}$$

$$40 \times 10^3 = \frac{2\pi \times 350 \times T}{60}$$

$$T = 1091.84 \text{ N-mt}$$

$$T_{max} = 1.25 \times 1091.34$$

$$= 1364.18 \text{ N-m}$$

$$T = \frac{\pi}{16} \times d^3 \times f_s$$

$$1364.18 \times 10^3 = \frac{\pi}{16} \times d^3 \times 30$$

$$d = 61.41 \text{ mm}$$

$$= 62 \text{mm}$$

2) Design of muff:

$$D = 2d + 13$$

$$= 2 \times 62 + 13 = 137$$

$$L = 3.5 \times 62 = 217 \text{ mm}$$

$$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$$

$$1364.18 \times 10^3 = \frac{\pi}{16} \times \frac{1374.62 \times f_s}{137}$$

$$f_s = 2.8 \text{ N/mm}$$

so, that ~~ta~~ calculated of f_s is less than given value the muff is safe.

3 design of key

$w = d/4 = 15.5$

$t = d/6 = 10.3$

$l = 3.5(d/2) = 108.5$

3 Design a Muff coupling to connect two shafts transmitting 100kw at 200rpm the permissible shearing and crushing stresses for the shaft and key materials are 50 N/mm² & 100N/mm² respectively. The material of muff is castiron with a permissible shear stress of 150N/mm². Assume that maximum torque transmitted is Equal to mean torque.

Sol N = 200rpm

P = 100kw

~~fs shaft muff = 50N/mm²~~

~~fs shaft = 100N/mm²~~

f_s shaft $= 50 N/mm^2$

f_c key $= 100 N/mm^2$

f_s shaft $= 15 N/mm^2$

1, Design of shaft

$$P = \frac{2\pi NT}{60}$$

$$100 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$T = 4774.64 \text{ N-m}$$

$$= \frac{\pi}{16} \times d^3 \times f_s$$

$$4774.64 = \frac{\pi}{16} \times d^3 \times 50$$

$$d^3 = 486.34$$

$$d = 78.6 \text{ mm}$$

$$\boxed{d \cong 80 \text{ mm}}$$

2, Design of muff

$$D = 2d + 13$$

$$= 2(80) + 13 = 173 \text{ mm}$$

$$L = 3.5 \times d$$

$$= 3.5 \times 80 = 280 \text{ mm}$$

$$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$$

$$4774.64 = \frac{\pi}{16} \times \frac{(173)^4 - (80)^4}{173} \times f_s$$

$$f_s = 4.92 \text{ N/mm}^2$$

3, Design of key:

$$\boxed{f_c = 2 \cdot f_s}$$

$$w = d/4$$

$$= 80/4 = 20 \text{mm}$$

$$t = 20 \text{mm}$$

$$l = \frac{3.5d}{2}$$

$$= 140$$

$$T = f_s \times l \times w \times \frac{d}{2}$$

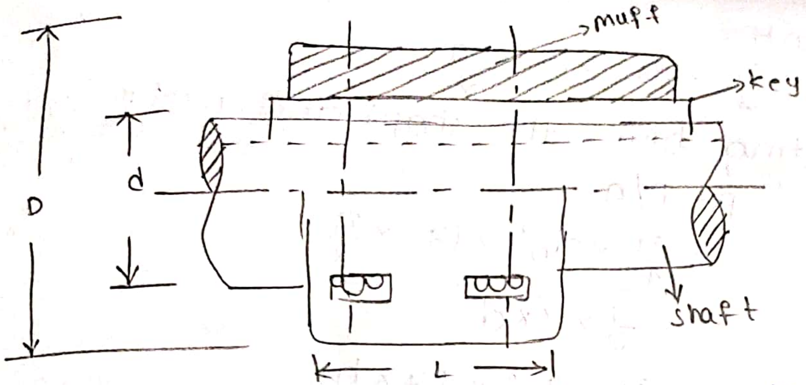
$$4774.64 \times 10^3 = f_s \times 140 \times 20 \times \frac{80}{2}$$

$$\boxed{f_s = 42.6}$$

$$4774.64 \times 10^3 = f_c \times 140 \times \frac{20}{2} \times \frac{80}{2}$$

$$\boxed{f_c = 85.2 \text{ N/mm}^2}$$

clamp or compression coupling: [split muff coupling]:



It is also known as split muff coupling in this case the muff (or) sleeve is made into two equal parts are bolted together. The two equal parts are the muff made of cast iron and the two parts are joined by means of a bolts. which are made with mild steel the no.of bolts may be two, four (or) six.

29/01/2020

(1) Design of shaft:

power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

By using this Eq.n Torque 'T' is calculated $T = \frac{\pi}{16} d^3 fs$

Here 'd' is calculated i.e dia of shaft.

(2) Design of sleeve (or) muff:

The muff is consider hallow shaft $D = 2d + 13$ & $L = 3.5d$

D = dia of hub

L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} fs$$

By using above Eq shear stress of hub is checked.

(3) Design of key:

$(w) = \frac{d}{4}$; $(t) = \frac{d}{6}$; $(L) = 3.5\frac{d}{2}$

Torque $(T) = f.s\,(L \times w) \times \frac{d}{2}$

By using this Eq.n shear stress is checked

Torque $(T) = fc \times (L \times \frac{t}{2}) \times \frac{d}{2}$

(4) Design of Bolt:

Force acting on the bolts.

$$F = \frac{\pi}{4} \times d_b^2 \times f_t$$

On each bolt

$$F = \frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}$$

Force acting b/w the shaft and muff

$$P = F/A$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d}$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times n}{4 \times L \times d}$$

Frictional force:

$$F_F = \mu . P . A$$

$$= \mu . \frac{\pi . d_b^2 f_t . n}{4 \times L \times d} \times \frac{1}{2} \pi d . L$$

$$= \mu . \frac{\pi^2 d_b^2 \times f_t . n}{8}$$

$\mu = \frac{F}{R_n}$

$F = \mu \times w$

$\boxed{F = \mu \times P \times}$

Torque

$$T = F_F \times \perp^{er} \text{distance}$$

$$= \frac{\mu . \pi^2 . d_b^2 . f_t \times n}{8} \times \frac{d}{2}$$

$$\boxed{T = \frac{\mu \times \pi^2 \times d_b^2 \times f_t \times n \times d}{16}}$$

1) Design a clamp coupling to transmit 30 kW power at 100 rpm. The allowable shear stress for the shaft and key is 40 M.Pa and the no of Bolts connecting to the two halves are 6. The permissiable tensile stress for the bolts is 70 MPa. The sufficient of friction b/w the muff & shaft surface may be taken as 0.3

$p = 30kW$

$N = 100 rpm$

f_s shaft $= 40 M.Pa$

f_s key $= 40 M.Pa$

$n = 6 \quad f_t = 70 MPa$

$\mu = 0.3$

i) Design of shaft:

$$P = \frac{2\pi NT}{60}$$

$$30\times10^3 = \frac{2\pi\times100\times T}{60}$$

$$T = 2864.7 \text{ N-m}$$

$$T_{max} = 1.25\times2864.7$$

$$T = \frac{\pi}{16}\times d^3\times f_s = 3580.9$$

$$2864.7\times10^3 = \frac{\pi}{16}\times d^3\times40.$$

$$\boxed{d = 71.6}$$

ii) $D = 2d+13$ $\qquad C = 3.5\times d$

$= 2(71.3)+13 \qquad = 3.5\times(71.3)$

$= 155.6 \qquad = 249.55$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$2864.7\times10^3 = \frac{\pi}{16}\times\frac{155.6^4-71.6^4}{155.6}\times f_s$$

$$\boxed{f_s = 4.05}$$

iii) $w = \frac{71.6}{4}$, $t = \frac{71.6}{6}$; $C = 3.5\left(\frac{71.6}{2}\right)$

$= 17.9 \qquad = 11.9 \qquad = 125.3$

$$T = f_s.(L\times w)\times\frac{d}{2}$$

$$2864.7\times10^3 = f_s\,(125.3\times17.9)\times\frac{71.6}{2}$$

$$\boxed{f_s = 35.6}$$

$$T = f_c\left(\left(L\times\frac{t}{2}\right)\times\frac{d}{2}\right)$$

$$2864.7\times10^3 = f_c\left(125.3\times\frac{11.9}{2}\right)\times\frac{71.6}{2}$$

$$\boxed{f_c = 107.3}$$

iv) $$T = \frac{\pi^2}{16}.\mu.d_b^2\times f_t\times n\times d$$

$$2864.7\times10^3 = \frac{\pi^2}{16}\times0.3\times d_b^2\times70\times6\times71.6$$

$$\boxed{d_b = 22.6}$$

Design a compression coupling for shaft to transmit 1300 N-m the allowable shear stress for the shaft and key 40 mPa and the no. of bolts connecting the two parts are the permissible tensile stress for the bolt material is 70 MPa. The coefficient friction is taken as 0.3

Sol. $\mu = 0.3$

$T = 1300$ N-m

f_s shaft, key $= 40 M.Pa$

f_t bolt $= 70 M.Pa$

$n = 4$

Shaft

$$T = \frac{\pi}{16}\times d^3\times f_s$$

$$1300\times10^3 = \frac{\pi}{16}\times d^3\times40$$

ii) Design of muff

$D = 2d+13 \qquad L = 3.5d$

$= 123 \qquad = 3.5\times55$

$\qquad = 192.5 = 193$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$1300\times10^3 = \frac{\pi}{16}\times\frac{123^4-55^4}{123}\times f_s$$

iii) Design of key:

$w=\frac{d}{4}=$ $\qquad l=3.5\left(\frac{d}{2}\right)$ $\qquad t=\frac{d}{6}$

$=13.75$ $\qquad =96.25$ $\qquad =9.1$

$T=f_s\times(l\times w)\times\frac{d}{2}$

$1300\times10^3= f_s(96.25\times13.75)\times\frac{55}{2}$

$\boxed{f_s=35.7}$

$T=f_c\left(l\times\frac{t}{2}\right)\times\frac{d}{2}$

$1300\times10^3= f_c\left(96.25\times\frac{9.1}{2}\right)\left(\frac{55}{2}\right)$

$\boxed{f_c=108}$

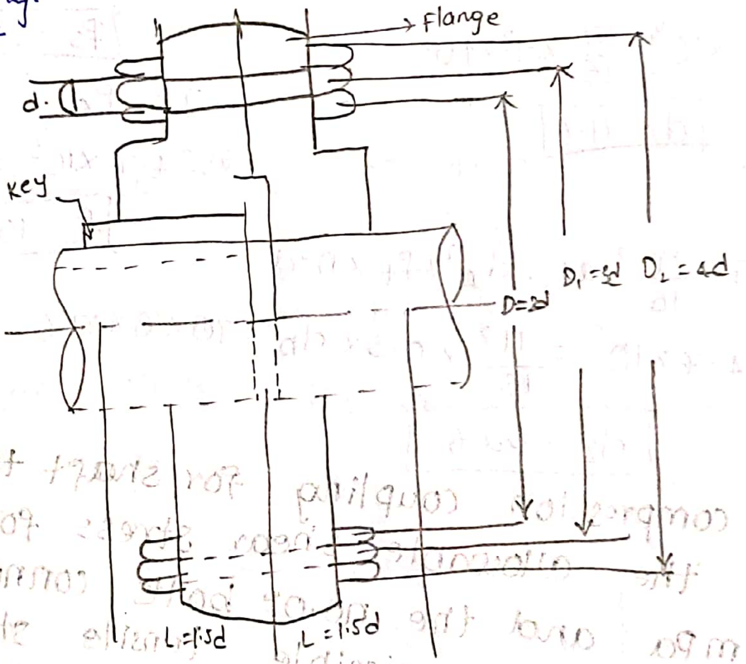
iv) Design of key:

$T=\frac{\pi^2}{16}\cdot\mu\cdot d_b^2\sqrt{t}\cdot n\cdot d$

$1300\times10^3=\frac{\pi^2}{16}(0.3)\,d_b^2\cdot70\cdot4$

$\boxed{d_s=21.3}$

Flange coupling:



d= diameter of the shaft

D= Diameter of the muff

D=2d

D_1 = pitch circle diameter of the flank

$D_1=3d$

D_2 = Diameter of the flange.

D_2 = 4d

T_f = Thickness of the flange

T_f = 0.5d

L = length of the flange

n = No. of the bolts
n = 3 for diameter upto 40mm
n = 4 for diameter upto 100mm
n = 6 for diameter upto 180mm

1. Design of shaft:

Power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

By using this Eq.n Torque 'T' calculated

$$T = \frac{\pi}{16 \cdot d^3 \cdot f_s}$$

Here 'd' is calculated is dia of shaft.

2. Design of muff:

The muff is consider n allow shaft

$D = 2d$ & $L = 1.5d$

D = dia of hub
L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

By using above Eq.n shear stress of hub is checked.

3. Design of Key:

Rectangle = d/6)

$w = \frac{d}{4}$; $t = \frac{d}{6}$; $l = L$

$$\text{Torque } (T) = f_s (l \times w) \times \frac{d}{2}$$

By using the Eq.n shear stress is checked

$$\text{Torque}(T) = f_c \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

By using this Eq.n crushing stress is calculated

4. Design of ~~Key~~ Flange:

Torque transmitted by the flange = circumference of the hub × thickness of the flange × Shear stress of the flange × radius of the hub.

$$T = \pi D \times t_f \times f_s \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times t_f \times f_s}$$

By using the above Eq.n shear stress is calculated.

The thickness of the protective circumferential of flange

$T_b = 0.25d$

Design for bolts: The total load of the bolt $= \frac{\pi}{4} d_b^2 . f_{sb}$

Torque transmitted $T = \frac{\pi}{4} d_b^2 . f_{sbolt} \times n \times \frac{D_1}{2}$

we can calculate diameter of bolt.

The crushing strength of the bolt $= n \times d_b \times t_f \times f_{cb}$

Torque transmitted $T = n \times t_f \times d_b \times f_{cb} \times \frac{D_1}{2}$

Design a cast iron protective type flange coupling to transmit 15KW at 900rpm from an electric motor two a compressor has 1.35 the following the service factor may be assume may be used. shear stress for the permissible stresses may be equal to 40 M.Pa crushing stress bolt shaft and key 80 M.Pa shear stress for cast iron for bolt and key 8M.Pa. Draw a neat sketch of the coupling.

sol: $P = 15KW$

$N = 900rpm$

$S.F = 1.35$

f_s bolt, shaft, key = 40M.Pa

f_c bolt & key = 80M.Pa

f_s muff & flange = 8M.Pa

1 * **shaft**

$$P = \frac{2\pi NT}{60}$$

$$15\times10^3 = \frac{2\pi \times 900 \times T}{60}$$

$T = \underline{159.15}$ N-m

$T_{max} = S.F \times T$

$T_{max} = 1.35 \times 159.15$

$= 214.85$ N-mt

$$T = \frac{\pi}{16} \times d^3 \times f_s$$

$d = 30.13$ mm $\cong 31$

$\boxed{n=3}$

2 * **Muff:**

$D = 2d = 2(31) = 62$

$L = 1.5d = 46.5 \cong 47$

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$$

$$159.15\times10^3 = \frac{\pi}{16} \frac{62^4 - 31^4}{62} \times f_s$$

$f_s = 4.89$ N/mm²

the design of muff is safe.

* **Key:**

$f_c = 2 \times f_s$

$f_s = 2 \times 4.89 =$

so hear taken sq. key

$\omega = t = \frac{d}{4} = 7.75 \cong 8$

$l = L = 1.5d = 46.5 \cong 47$

$$T = f_s \times (l \times \omega) \times \frac{d}{2}$$

$f_s = \underline{36.85}$ N/mm²

$$T = f_c \times (l \times \frac{t}{2}) \times \frac{d}{2}$$

f [illegible] N/mm²

iii. **flange:**

$D_1 = 3d =$

$t_f = 0.5d =$

$t_p = 0.25d$

$T = \pi D \times t_f \times f_s \times D/2$

$f_s = 2.22$ N/mm²

iv. **Bolt**

$T = \frac{\pi}{4} \times d_b^2 \times n \times f_s \times \frac{D_1}{2}$

$d_b =$

$T = n \cdot t_f \times d_b \times f_c \times \frac{D}{2}$

Diagram

ii. Design a cast iron flang coupling to connect two shafts in order to transmit 7.5 kW at 720 rpm the following permissible stress may be assumed permissible shear stress for shaft bolt and key equal to 33 N/mm² permissible crushing stress for bolt and key 60 N/mm². The permissible shear stress for cast iron 15 N/mm².

$P = 7.5$ kW

$N = 720$ rpm

f_s shaft, bolt key $= 33$ N/mm²

f_c bolt key $= 60$ N/mm²

f_s muff, flang $= 15$ N/mm²

shaft:

$$P = \frac{2\pi N T}{60}$$

$$\frac{7.5 \times 10^3 \times 60}{2\pi \times 720} = T$$

$T = 99.4$ N-m

$T_{max} = SF \times T$

$$T = \frac{\pi}{16} \times d^3 \times F_s$$

$$d^3 = \frac{T \times 16}{\pi \times F_s}$$

$$d^3 = \frac{99.4 \times 10^3 \times 16}{\pi \times 33}$$

$d = 24.8$ mm $\simeq 25$

$\boxed{n = 3}$

Muff

$D = 2d = 2(25) = 50$

$L = 1.5d = 1.5(25) = 37.5 \simeq 38$

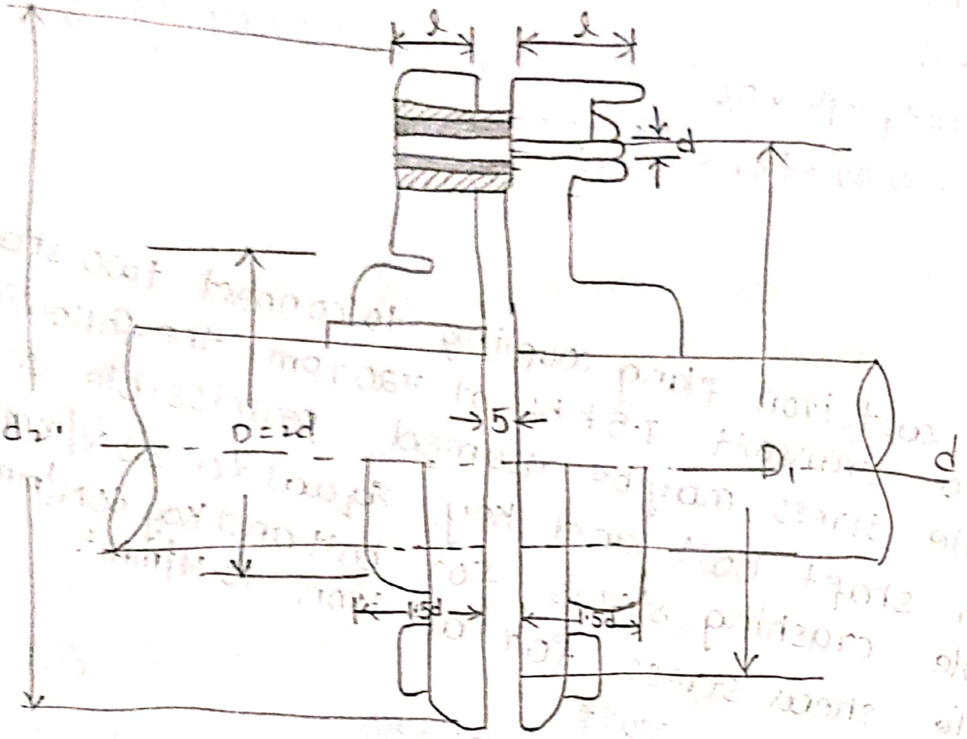
$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$$

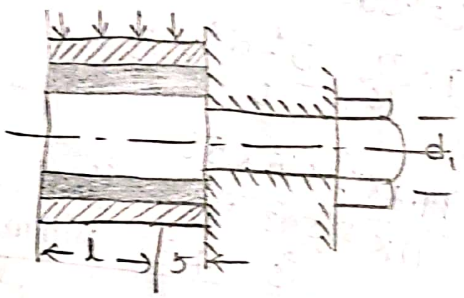
$$\frac{99.4 \times 10^3 \times 16 \times 50}{\pi \times 5859375} = f_s$$

$\boxed{f_s = 4.31 \text{ N/mm}^2}$

Flexible flange coupling:

Bushed pin flexible coupling:





A Bushed pin flexible coupling is a modification of rigid type flange coupling. the bolts of a coupling are known as pins. The rubber or leather bushes are used over the pins. The two parts of the coupling are disimillar in construction. A clearance of 5mm is left b/w the two parts of the couplings There is no rigid connection b/w them and the drive takes place through the medium of compressible rubber or leather bushes.

l = Length of the bush in the flange.
d_2 = diameter of the bush
P_b = bearing pressure on the bush or pin

D_1 = pitch circle diameter of the pins.

d_1 = diameter of pin $d_1 = \frac{0.5d}{\sqrt{n}}$

over all diameter of rubber bush

$$d_2 = d_1 + 4 + 2\times2 + 2\times6 = d_1 + 20$$

Diameter of pitch circle of the pins

$$D_1 = 2\times d + d_2 + 2\times6$$

Bearing load acting on each pin

$$W = P_b \times d_2 \times l \quad = 32l.$$

Total Bearing load on the bush or pin.

$$= W\times n$$

$$= P_b \times d_2 \times l \times n$$

Maximum Torque" transmitted by the coupling

$$T = W \times n \times \frac{D_1}{2}$$

Direct shear stress give to pure torsion in the coupling fails

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2}$$

The bush portion of the pin act as a cantilever beam of length 'l'. assume a uniform distributed load 'W' along the bush the bending moment

$$M = W\left(\frac{l}{2} + 5\right)$$

The bending stress = $\sigma = \frac{M}{Z}$

$$Z = \frac{\pi}{32} d_1^3$$

since, the pin is subjected to bending stress and shear stress. Therefore, the design must be checked either for maximum principle stress or maximum shear stress.

Max principle stress = $\frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\tau^2}\right)$

Max shear stress = $\frac{1}{2}\left[\sqrt{\sigma_1^2 + 4\tau^2}\right]$.

The value of Maximum principle stresses varies from 28 - 42 M.Pa

1) **Design of hub(or) muff :**

The muff is considered hallowshaft

$D=2d$ & $L=1.5d$

D= diameter of hub

L= length of hub

$T=\frac{\pi}{16}\ \frac{D^4-d^4}{D}\cdot f_s$

By using above Eq.n shear stress of hub is checked

2) **Design of key:**

$w=\frac{d}{4}$; $t=\frac{d}{6}$; $l=L$

$\text{Torque}(T)=f_s(l\times w)\times\frac{d}{2}$

By using the above Eq.n shear stress is calculated

$\text{Torque}(T)=f_c\times(l\times\frac{t}{2})\times\frac{d}{2}$

By using this Eq.n crushing stress is calculated.

3) **Design of flange:**

Torque transmitted by the flange = circumference of hub× thickness of flange× shear stress of flange× radius of hub

$T=\pi D\times t_f\times f_s\times\frac{D}{2}$

$$\boxed{T=\frac{\pi D^2}{2}\times t_f\times f_s}$$

By using above Eq.n shear stress is calculated.

The thickness of protective circumfrential flange $\boxed{T_b=0.25d}$

(1, Design a bush pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32kW at 960 rpm the overall torque is 20% more than the mean torque. The material properties are as follows.

i, The allowable shear stress and crushing stress for shaft and key material is 40 M.Pa & 80 M.Pa

ii, The allowable shear stress for cast iron is 15 M.Pa

iii, The allowable bearing pressure for rubber bush 0.8 N/m

The material for the pin is same as shaft and key. Draw a neat sketch of coupling.

Given,

$P = 32 KW$

$N = 960$

$T_{max} = 1.20\ T_{mean}$

f_s key $= 40 M.Pa$

f_c key $= 80 M.Pa$

$P_b = 0.8 N/mm^2$

f muff & flange $= 15 M.Pa$

Design of pin:

$$P = \frac{2\pi N T}{60}$$

$$32\times10^3 = \frac{2\pi\times960\times T}{60}$$

$$\boxed{T = 318.30}$$

$$T_{max} = 1.20 \times T_{mean}$$

$$T_{max} = 1.20 \times 318.30$$

$$\boxed{T_{max} = 381.96\ N\text{-}m}$$

$$T_{max} = \frac{\pi}{16}\times d^3 \times f_s$$

shaft

$$381.96\times10^3 = \frac{\pi}{16}\times d^3 \times 40$$

$$\boxed{d^3 = 36.5} \cong \cancel{8}\ 40$$

$$d_1 = \frac{0.5\times d}{\sqrt{n}} \qquad n = 6$$

$$d_1 = \frac{0.5\times 40}{\sqrt{6}}$$

$$\boxed{d_1 = 8.1} \cong 20\ mm$$

$$d_2 = d_1 + 4 + 2\times2 + 2\times6$$

$$d_2 = 20 + 4 + 2\times2 + 2\times6$$

$$\boxed{d_2 = 40}\ mm$$

$$D_1 = 2\times d + d_2 + 2\times6$$

$$D_1 = 2\times40 + 40 + 2\times6$$

$$\boxed{D_1 = 132 mm}$$

$$W = P_b \times d_2 \times l$$

$$W = 0.8 \times 40 \times l$$

$$\boxed{W = 32\,l}$$

$$T_{max} = W \times n \times \frac{D_1}{2}$$

$$\overset{381.96\times10^3}{\cancel{T_m}} = 32\,l \times 6 \times \frac{132}{2}$$

$$l = 30.14\ mm \cong 32$$

$$W = 32\times30.14$$

$$\boxed{W = 964.48}\ N$$

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2}$$

$$\tau = \frac{964.48}{\frac{\pi}{4}(\overset{20}{\cancel{8.1}})^2} = 3.07$$

$$M = W\left(\frac{l}{2} + 5\right)$$

$$= 964.48\left(\frac{\overset{32}{\cancel{30.14}}}{2} + 5\right)$$

$$= 20254.08\ N\text{-}mm$$

$$\sigma = \frac{M}{Z} \qquad Z = \frac{\pi}{32\cancel{d^3}} d_1^3$$

$$Z = \frac{\pi\ (20^3)}{32\times\cancel{(20)}^3} = 1.227\times10^{-5}$$

$$\sigma = \frac{20254.08}{\underset{785.3}{\cancel{1.227\times10^{-5}}}} = 25.79$$

$$\text{Max. principle stress} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[25.79 + \sqrt{(25.79)^2 + 4(3.07)^2}\right]$$

$$= 25.9 \cong 30$$

Max. shear stress =

$$\frac{1}{2}\left[\sigma^2 + 4\tau^2\right]$$

$$= \frac{1}{2}\left[(25.79)^2 + 4(3.07)^2\right]$$

$$= 351.41$$

ⅱ) Design of hub:-

$D = 2d$

$D = 2(40) = 80$

$l = 1.5d$

$l = (1.5)(40)$

$= 60$

$$T = \frac{\pi}{16} \cdot \frac{D^4 - d^4}{D} \cdot fs$$

$$= \frac{\pi}{\cancel{4}} \frac{80^4 - 40^4}{\cancel{80}_{20}} \cdot \cancel{40}$$

$$= 1507964\cancel{4.74}$$

Design of Key:

$$\omega = \frac{d}{4} = \frac{40}{4} = 10$$

$$T = fs \times (l \times \omega) \times \frac{d}{2}$$

$$= 40 \times (32 \times 964.48) \times 20$$

$$= 24690688$$

$$T = fc \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

$$= 80 \times (32 \times$$

27/02/2020

UNIT - 5

COUPLINGS

coupling is a device For connecting the ends of two shafts together.

Types of couplings: couplings are two types.

(1) Rigid coupling
(2) Flexible coupling.

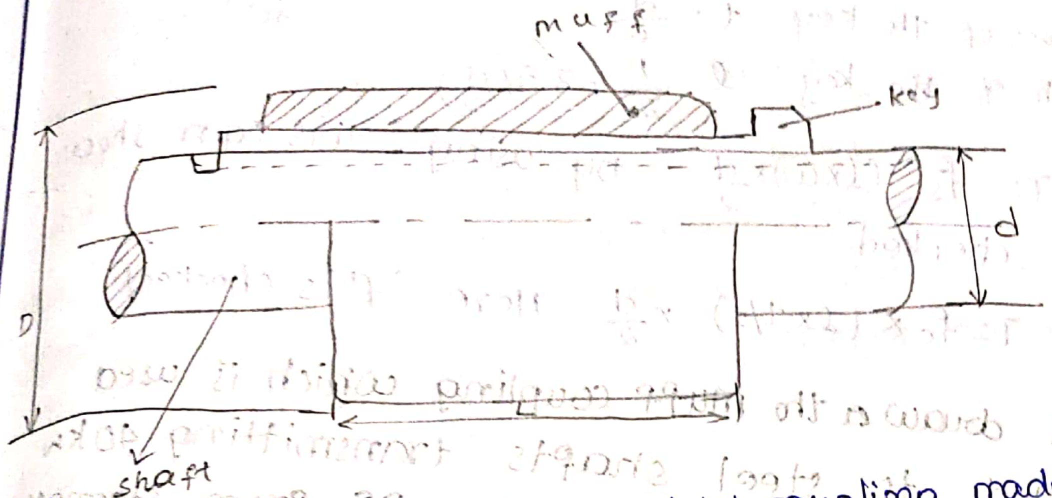
Rigid coupling: It is used to connect two shafts which are perfectly alingned i.e collinear. These couplings donot permit any mis alignment of shafts that common forms of rigid coupling are

(1) Muff coupling.
(2) split muff (or) clamp (or) compression coupling.
(3) Flange coupling.

Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment i.e axis are not colinear. These couplings permit mis alignment and posses flexibility The common forms of flexible couplings are.

(i) Hold ham couplings (permits small lateral mis alignment).
(ii) universal couplings (permits small angular miss alignment).
(iii) Bushed pin type coupling (absorbed shafts and permits small amount of angular and lateral mis alignment).

Design of muff coupling (sleeve coupling):



27/01/2020

It is the simplest type of rigid coupling made of cast iron it consists of a hallow cylinder whose inner diameter is the same as that of the shaft it is fitted over the ends of the two shafts by means of a "Gib head key."

Design of shaft: power transmitted by the shaft torque 'T' is

$P = \frac{2\pi N T}{60}$ by using this eq.n

calculated

Torque $T = \cancel{\frac{\pi}{16 d^3 F_s} d^3}$ $\quad T = \frac{\pi}{16 . d^3 . F_s}$

Heare d is calculated i.e is diameter of the shaft.

Design of sleeve (or) muff: The muff is consider hallow shaft.

$D = 2d + 13$ & $L = 3.5 d$

D = diameter of the hub

L = length of the hub.

$\because F_s$ = shear stress.

Torque, $T = \frac{\pi}{16} \frac{(D^4 - d^4)}{d} . F_s$

By using the above eq.n shear stress of hub is checked

Design of key:

[f = stress]

f_c = crushing stress

width of the key $w = \frac{d}{64}$

Thickness of the key $t = \frac{d}{6}$

length of the key $l = \frac{L}{2} = 3.5\left(\frac{d}{2}\right)$

Torque $T = f_s \cdot (l \times w) \times \frac{d}{2}$ by using this eq.n shear stress is checked.

Torque $T = f_c \times (l \times t/2) \times \frac{d}{2}$ Here 'f' is checked

1) Design and draw a the muff coupling which is used to connected to steel shafts transmitting 40kw at 350rpm. Design shaft and muff from strength point of view and other dimensions by emphirical forumales shear stress for muff and shaft 15 N/mm² and 30 N/mm². Assume maximum torque to be 25% more than mean torque.

sol N = 350rpm

P = 40kw

f_s muff = 15 N/mm²

f_s shaft = 30N/mm²

$T_{max} = 1.25 \times T_{mean}$

(1) Design of shaft:

$$P = \frac{2\pi N T}{60}$$

$$40 \times 10^3 = \frac{2\pi \times 350 \times T}{60}$$

$$\underline{T = 1091.84 \text{ N-mt}}$$

$T_{max} = 1.25 \times 1091.34$

$= 1364.18$ N-m

$$T = \frac{\pi}{16} \times d^3 \times f_s$$

$$1364.18 \times 10^3 = \frac{\pi}{16} \times d^3 \times 30$$

$$\underline{d = 61.41 \text{ mm}}$$

$= 62$mm

(2) Design of muff:

$D = 2d + 13$

$= 2 \times 62 + 13 = 137$

$L = 3.5 \times 62 = 217$ mm

$$T = \frac{\pi}{16} \times \frac{D^4 - d^4}{D} \times f_s$$

$$1364.18 \times 10^3 = \frac{\pi}{16} \times \frac{1374.62}{137} \times f_s$$

$$\underline{f_s = 2.8 \text{ N/mm}}$$

so, that calculated of f_s is less than given value the muff is safe.

3) design of key

$w = d/4 = 15.5$

$t = d/6 = 10.3$

$l = 3.5(d/2) = 108.5$

3) Design a Muff coupling to connect two shafts transmitting 100kw at 200rpm the permissible shearing and crushing stresses for the shaft and key materials are 50 N/mm² & 100N/mm² respectively. The material of muff is castiron with a permissible shear stress of 15N/mm². Assume that maximum torque transmitted is Equal to mean torque.

Sol: $N = 200\text{rpm}$

$P = 100\text{kw}$

~~fs muff = 50N/mm²~~

~~fs shaft = 100N/mm²~~

f_s shaft $= 50\text{N/mm}^2$

f_c key $= 100\text{N/mm}^2$

f_s shaft $= 15\text{N/mm}^2$

1, Design of shaft

$$P = \frac{2\pi NT}{60}$$

$$100\times10^3 = \frac{2\pi\times200\times T}{60}$$

$$T = 4774.64 \text{ N-m}$$

$$= \frac{\pi}{16}\times d^3\times f_s$$

$$4774.64 = \frac{\pi}{16}\times d^3\times 50$$

$$d^3 = 486.34$$

$$d = 78.6 \text{ mm}$$

$$\boxed{d \simeq 80\text{ mm}}$$

2, Design of muff

$$D = 2d + 13$$

$$= 2(80) + 13 = 173\text{mm}$$

$$L = 3.5\times d$$

$$= 3.5\times80 = 280\text{ mm}$$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$4774.64 = \frac{\pi}{16}\times\frac{(173)^4-(80)^4}{173}\times f_s$$

$$f_s = 4.92\text{ N/mm}^2$$

3, Design of key:

$$\boxed{f_c = 2\cdot f_s}$$

$$w = d/4$$

$$= 80/4 = 20\text{mm}$$

$$l = \frac{3.5d}{2}$$

$$= 140$$

$$t = 20\text{mm}$$

$$T = f_s\times l\times w\times\frac{d}{2}$$

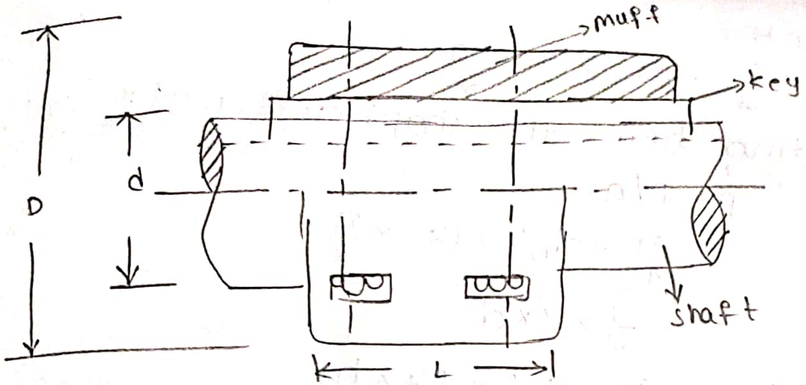
$$4774.64\times10^3 = f_s\times140\times20\times\frac{80}{2}$$

$$\boxed{f_s = 42.6}$$

$$4774.64\times10^3 = f_c\times140\times\frac{20}{2}\times\frac{80}{2}$$

$$\boxed{f_c = 85.2\text{ N/mm}^2}$$

clamp or compression coupling: [split muff coupling]:



It is also known as split muff coupling in this case the muff (or) sleeve is made into two equal parts the muff made of cast iron and the two parts are bolted together. The two equal parts are joined by means of a bolts. which are made with mild steel the no.of bolts may be two, four (or) six.

29/01/2020

(1) Design of shaft:

power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

By using this Eq.n Torque 'T' is calculated $T = \frac{\pi}{16 d^3 fs}$

Here 'd' is calculated i.e dia of shaft.

(2) Design of sleeve (or) muff:

The muff is consider hallow shaft $D = 2d + 13$ & $l = 3.5d$

D = dia of hub

l = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} fs$$

By using above Eq shear stress of hub is checked.

(3) Design of key:

$(w) = \frac{d}{4}$; $(t) = \frac{d}{6}$; $(L) = 3.5\frac{d}{2}$

$$\text{Torque } (T) = f.s\,(L \times w) \times \frac{d}{2}$$

By using this Eq.n shear stress is checked

$$\text{Torque } (T) = fc \times (L \times \frac{t}{2}) \times \frac{d}{2}$$

(4) Design of Bolt:

Force acting on the bolts.

$$F = \frac{\pi}{4} \times d_b^2 \times f_t$$

On each bolt

$$F = \frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}$$

Force acting b/w the shaft and muff

$$P = F/A$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times \frac{n}{2}}{\frac{1}{2} \times L \times d}$$

$$= \frac{\frac{\pi}{4} \times d_b^2 \times f_t \times n}{4 \times L \times d}$$

Frictional force:

$$F_F = \mu . P . A$$

$$= \mu . \frac{\pi . d_b^2 f_t . n}{4 \times L \times d} \times \frac{1}{2} \pi d . L$$

$$= \mu . \frac{\pi^2 d_b^2 \times f_t . n}{8}$$

$\mu = \frac{F}{R_n} =$

$F = \mu \times W$

$F = \mu \times P \times$

Torque

$$T = F_F \times \perp \text{distance}$$

$$= \frac{\mu . \pi^2 . d_b^2 . f_t \times n}{8} \times \frac{d}{2}$$

$$\boxed{T = \frac{\mu \times \pi^2 \times d_b^2 \times f_t \times n \times d}{16}}$$

1) Design a clamp coupling to transmit 30 kW power at 100 rpm The allowable shear stress for the shaft and key is 40 M.Pa and the no of Bolts connecting to the two halves are 6. The permissiable tensile stress for the bolts is 70 MPa The sufficient of friction b/w the muff & shaft surface may be taken as 0.3

$p = 30\,kW$
$N = 100\,rpm$
f_s shaft $= 40\,M.Pa$
f_s key $= 40\,M.Pa$
$n = 6$ $\quad f_t = 70\,MPa$
$\mu = 0.3$

i, Design of shaft:

$$P = \frac{2\pi NT}{60}$$

$$30\times10^3 = \frac{2\pi\times100\times T}{60}$$

$T = 2864.7\ N\text{-}m$

$T_{max} = 1.25\times2864.7$

$$T = \frac{\pi}{16}\times d^3\times f_s = 3580.9$$

$$2864.7\times10^3 = \frac{\pi}{16}\times d^3\times40.$$

$$\boxed{d = 71.6}$$

ii, $D = 2d+13$ $\quad C = 3.5\times d$
$= 2(71.3)+13$ $\quad = 3.5\times(71.3)$
$= 155.6$ $\quad = 249.55$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$2864.7\times10^3 = \frac{\pi}{16}\times\frac{155.6^4-71.6^4}{155.6}\times f_s$$

$$\boxed{f_s = 4.05}$$

iii, $\omega = \frac{71.6}{4}$, $t = \frac{71.6}{6}$; $C = 3.5\left(\frac{71.6}{2}\right)$
$= 17.9$ $\quad = 11.9$ $\quad = 125.3$

$$T = f_s.(L\times\omega)\times\frac{d}{2}$$

$$2864.7\times10^3 = f_s(125.3\times17.9)\times\frac{71.6}{2}$$

$$\boxed{f_s = 35.6}$$

$$T = f_c\left(L\times\frac{t}{2}\right)\times\frac{d}{2}$$

$$2864.7\times10^3 = f_c\left(125.3\times\frac{11.9}{2}\right)\times\frac{71.6}{2}$$

$$\boxed{f_c = 107.3}$$

iv, $T = \frac{\pi^2}{16}\cdot\mu\cdot d_b^2\times f_t\times n\times d$

$$2864.7\times10^3 = \frac{\pi^2}{16}\times0.3\times d_b^2\times70\times6\times71.6$$

$$\boxed{d_b = 22.6}$$

Design a compression coupling for shaft to transmit 1300 N-m the allowable shear stress for the shaft and key 40 mPa and the no.of bolts connecting the two parts are the permissible tensile stress for the bolt material is 70 MPa. The coefficient friction is taken as 0.3

Sol. $\mu = 0.3$
$T = 1300\ N\text{-}m$
f_s shaft, key $= 40\,M.Pa$
f_t bolt $= 70\,M.Pa$
$n = 4$

Shaft

$$T = \frac{\pi}{16}\times d^3\times f_s$$

$$1300\times10^3 = \frac{\pi}{16}\times d^3\times40$$

ii, Design of muff

$D = 2d+13$ $\quad C = 3.5d$
$= 123$ $\quad = 3.5\times55$
$\quad = 192.5 = 193$

$$T = \frac{\pi}{16}\times\frac{D^4-d^4}{D}\times f_s$$

$$1300\times10^3 = \frac{\pi}{16}\times\frac{123^4-55^4}{123}\times f_s$$

iii, **Design of key:**

$w = \frac{d}{4} =$ $= 13.75$

$l = 3.5\left(\frac{d}{2}\right)$ $= 96.25$

$t = \frac{d}{6}$ $= 9.1$

$T = fs \times (L \times w) \times \frac{d}{2}$

$1300 \times 10^3 = fs(96.25 \times 13.75) \times \frac{55}{2}$

$\boxed{fs = 35.7}$

$T = fc\left(L \times \frac{t}{2}\right) \times \frac{d}{2}$

$1300 \times 10^3 = fc\left(96.25 \times \frac{9.1}{2}\right)\left(\frac{55}{2}\right)$

$\boxed{fc = 108}$

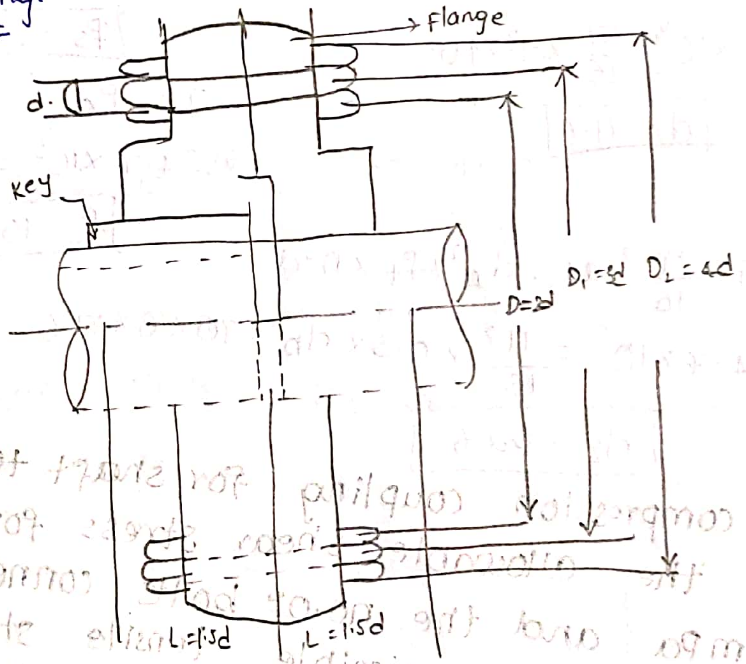
iv, **Design of key:**

$T = \frac{\pi^2}{16} \cdot \mu \cdot d_b^2 \cdot \sqrt{t} \cdot n \cdot d$

$1300 \times 10^3 = \frac{\pi^2}{16}(0.3)\, d_b^2 \cdot 70 \cdot 4$

$\boxed{d_s = 21.3}$

Flange coupling:



d = diameter of the shaft

D = Diameter of the muff

D = 2d

D_1 = pitch circle diameter of the flank

$D_1 = 3d$

D_2 = Diameter of the flange.

$D_2 = 4d$

T_f = Thickness of the flange

$T_f = 0.5d$

L = length of the flange

n= No.of the bolts

n=3 for diameter upto 40mm

n=4 for diameter uplo 100mm

n=6 for diameter upto 180mm

1, **Design of shaft:**

Power transmitted by shaft

$$P = \frac{2\pi N T}{60}$$

By using this Eq.n Torque 'T' calculated

$$T = \frac{\pi}{16 . d^3 . f_s}$$

Here 'd' is calculated is dia of shaft.

2, **Design of muff:**

The muff is consider hallow shaft

$D = 2d$ & $L = 1.5d$

D = dia of hub

L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

By using above Eq.n shear stress of hub is checked.

3, **Design of Key:**

Rectangle = d/6)

$w = \frac{d}{4}$; $t = \frac{d}{6}$; $l = L$

Torque (T) = $f_s (l \times w) \times \frac{d}{2}$

By using the Eq.n shear stress is checked

Torque(T) = $f_c \times (l \times \frac{t}{2}) \times \frac{d}{2}$

By using this Eq.n crushing stress is calculated

4, **Design of ~~Key~~ Flange:**

Torque transmitted by the flange = circumference of the hub × thickness of the flange × shear stress of the flange × radius of the hub.

$$T = \pi D \times t_f \times f_s \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times t_f \times f_s}$$

By using the above Eq.n shear stress is calculated.

The thickness of the protective circumfrential of flange

$T_b = 0.25d$

Design for bolts: The total load of the bolt $= \frac{\pi}{4} d_b^2 \cdot f_{sb}$

Torque transmitted $T = \frac{\pi}{4} d_b^2 \cdot f_{sb} \times n \times \frac{D_1}{2}$

we can calculate diameter of bolt.

The crushing strength of the bolt $= n \times d_b \times t_f \times f_{cb}$

Torque transmitted $T = n \times t_f \times d_b \times f_{cb} \times \frac{D_1}{2}$

Design a cast iron protective type flange coupling to transmit 15KW at 900rpm from an electric motor two a compressor has 1.35 the following the service factor may be assume may be used. shear stress for the permissible stresses may be used equal to 40 M.Pa crushing stress bolt shaft and key 80 M.Pa shear stress for cast iron for bolt and key. Draw a neat sketch of the coupling. 8M.Pa

Sol: $P = 15KW$

$N = 900rpm$

$S.F = 1.35$

f_s bolt, shaft, key = 40M.Pa

f_c bolt & key = 80M.Pa

f_s muff & flange = 8M.Pa

* **Shaft**

$P = \frac{2\pi NT}{60}$

$15\times10^3 = \frac{2\pi \times 900 \times T}{60}$

$T = \underline{159.15}$ N-m

$T_{max} = S.F \times T$

$T_{max} = 1.35 \times 159.15$

$= 214.85$ N-mt

$T = \frac{\pi}{16} \times d^3 \times f_s$

$d = 30.13\ mm \cong 31$

$\boxed{n=3}$

* **Muff:**

$D = 2d = 2(31) = 62$

$L = 1.5d = 46.5 \cong 47$

$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$

$159.15\times10^3 = \frac{\pi}{16} \frac{62^4 - 31^4}{62} \times f_s$

$f_s = 4.89\ N/mm^2$

the design of muff is safe.

* **Key:**

$f_c = 2 \times f_s$

$f_s = 2 \times 4.89 =$

so hear taken sq.key

$\omega = t = \frac{d}{4} = 7.75 \cong 8$

$l = L = 1.5d = 46.5 \cong 47$

$T = f_s \times (l \times \omega) \times \frac{d}{2}$

$f_s = \underline{36.85}\ N/mm^2$

$T = f_c \times (l \times \frac{t}{2}) \times \frac{d}{2}$

f [illegible] N/mm^2

i. **Flange:**

$D_1 = 3d =$

$t_f = 0.5d =$

$t_p = 0.25d$

$T = \pi D \times t_f \times f_s \times D/2$

$f_s = 2.22$ N/mm²

4. **Bolt**

$T = \frac{\pi}{4} \times d_b^2 \times n \times f_s \times \frac{D_1}{2}$

$d_b =$

$T = n \cdot t_f \times d_b \times f_c \times \frac{D}{2}$

Diagram

2. Design a cast iron flang coupling to connect two shafts in order to transmit 7.5 kW at 720 rpm the following permissible stress may be assumed permissible shear stress for shaft bolt and key equal to 33 N/mm² permissible crushing stress for bolt and key 60 N/mm². The permissible shear stress for cast iron 15 N/mm².

$P = 7.5$ kW

$N = 720$ rpm

f_s shaft, bolt key $= 33$ N/mm²

f_c bolt key $= 60$ N/mm²

f_s muff, flang $= 15$ N/mm²

Shaft:

$P = \frac{2\pi NT}{60}$

$\frac{7.5 \times 10^3 \times 60}{2\pi \times 720} = T$

$T = 99.4$ N-m

$T_{max} = SF \times T$

$T = \frac{\pi}{16} \times d^3 \times f_s$

$d^3 = \frac{T \times 16}{\pi \times f_s}$

$d^3 = \frac{99.4 \times 10^3 \times 16}{\pi \times 33}$

$d = 24.8 \text{mm} \simeq 25$

$\boxed{n = 3}$

Muff

$D = 2d = 2(25) = 50$

$L = 1.5d = 1.5(25) = 37.5 \simeq 38$

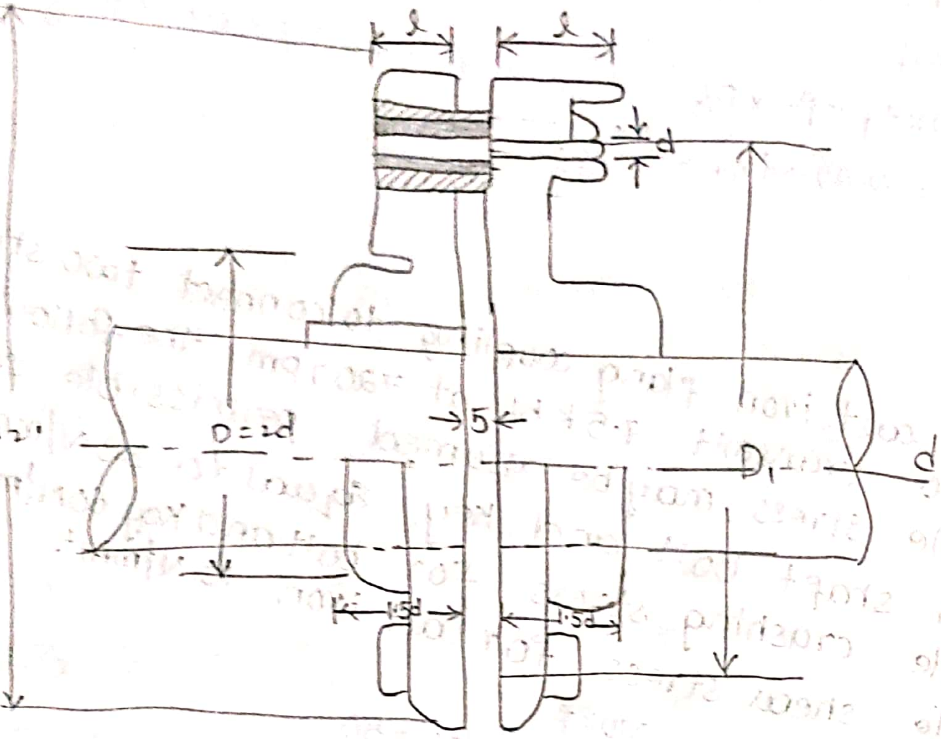
$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \times f_s$

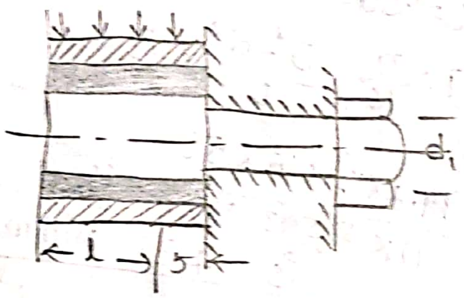
$\frac{99.4 \times 10^3 \times 16 \times 50}{\pi \times 5859375} = f_s$

$\boxed{f_s = 4.31 \text{ N/mm}^2}$

Flexible flange coupling:

Bushed pin flexible coupling:





A Bushed pin flexible coupling is a modification of rigid type flange coupling. the bolts of a coupling are known as pins. The rubber or leather bushes are used over the pins. The two parts of the coupling are disimillar in construction. A clearance of 5mm is left b/w the two parts of the couplings There is no rigid connection b/w them and the drive takes place through the medium of compressible rubber or leather bushes.

l= Length of the bush in the flange.

d_2= diameter of the bush

P_b = bearing pressure on the bush or pin

D_1 = pitch circle diameter of the pins.

d_1 = diameter of pin $d_1 = \frac{0.5d}{\sqrt{n}}$

overall diameter of rubber bush

$$d_2 = d_1 + 4 + 2 \times 2 + 2 \times 6 = d_1 + 20$$

Diameter of pitch circle of the pins

$$D_1 = 2 \times d + d_2 + 2 \times 6$$

Bearing load acting on each pin

$$W = P_b \times d_2 \times l \quad = 32l.$$

Total Bearing load on the bush or pin.

$$= W \times n$$

$$= P_b \times d_2 \times l \times n$$

Maximum Torque" transmitted by the coupling

$$T = W \times n \times \frac{D_1}{2}$$

Direct shear stress give to pure torsion in the coupling fails

$$\tau = \frac{W}{\frac{\pi}{4} d_1^2}$$

The bush portion of the pin act as a cantilever beam of length 'l'. assume a uniform distributed load 'W' along the bush the bending moment

$$M = W\left(\frac{l}{2} + 5\right)$$

The bending stress = $\sigma_1 = \frac{M}{Z}$

$$Z = \frac{\pi}{32} d_1^3$$

since, the pin is subjected to bending stress and shear stress. Therefore, the design must be checked either for maximum principle stress or maximum shear stress.

Max principle stress = $\frac{1}{2}\left(\sigma \mp \sqrt{\sigma^2 + 4\tau^2}\right)$

Max shear stress = $\frac{1}{2}\left[\sqrt{\sigma_1^2 + 4\tau^2}\right]$.

The value of Maximum principle stresses varies fro
28 - 42 M.Pa

1) **Design of hub(or) muff :**

The muff is considered hallowshaft

$D = 2d$ & $L = 1.5d$

D = diameter of hub
L = length of hub

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

By using above Eq.n shear stress of hub is checked

2) **Design of key:**

$\omega = \frac{d}{4}$; $t = \frac{d}{6}$; $l = L$

$$\text{Torque}(T) = f_s (l \times w) \times \frac{d}{2}$$

By using the above Eq.n shear stress is calculate

$$\text{Torque}(T) = f_c \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

By using this Eq.n crushing stress is calculated.

3) **Design of flange:**

Torque transmitted by the flange = circumference of hub × thickness of flange × shear stress of flange × radius of hub

$$T = \pi D \times t_f \times f_s \times \frac{D}{2}$$

$$\boxed{T = \frac{\pi D^2}{2} \times t_f \times f_s}$$

By using above Eq.n shear stress is calculated.

The thickness of protective circumferential flange

$$\boxed{T_b = 0.25d}$$

1, Design a bush pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 rpm the overall torque is 20% more than the mean torque. The material properties are as follows.

i, The allowable shear stress and crushing stress for shaft and key material is 40 M.Pa & 80 M.Pa

ii, The allowable shear stress for cast iron is 15 M.Pa

iii, The allowable bearing pressure for rubber bush 0.8 N/m

The material for the pin is same as shaft and key. Draw a neat sketch of coupling.

Sol. Given,

$P = 32\,kW$

$N = 960$

$T_{max} = 1.20\,T_{mean}$

$f_s\ key = 40\,M.Pa$

$f_c\ key = 80\,M.Pa$

$P_b = 0.8\,N/mm^2$

$f\ muff\ \&\ flange = 15\,M.Pa$

Design of pin:

$$P = \frac{2\pi N T}{60}$$

$$32\times10^3 = \frac{2\pi\times960\times T}{60}$$

$$\boxed{T = 318.30}$$

$$T_{max} = 1.20 \times T_{mean}$$

$$T_{max} = 1.20 \times 318.30$$

$$\boxed{T_{max} = 381.96\ N\text{-}m}$$

shaft

$$T_{max} = \frac{\pi}{16}\times d^3 \times f_s$$

$$381.96\times10^3 = \frac{\pi}{16}\times d^3\times 40$$

$$\boxed{d^3 = 36.5} \cong \cancel{8}\ 40$$

$$d_1 = \frac{0.5\times d}{\sqrt{n}} \qquad n = 6$$

$$d_1 = \frac{0.5\times40}{\sqrt{6}}$$

$$\boxed{d_1 = 8.1} \cong 20\ mm$$

$$d_2 = d_1 + 4 + 2\times2 + 2\times6$$

$$d_2 = 20 + 4 + 2\times2 + 2\times6$$

$$\boxed{d_2 = 40}\ mm$$

$$D_1 = 2\times d + d_2 + 2\times6$$

$$D_1 = 2\times40 + 40 + 2\times6$$

$$\boxed{D_1 = 132\,mm}$$

$$W = P_b \times d_2 \times l$$

$$W = 0.8\times40\times l$$

$$\boxed{W = 32\,l}$$

$$T_{max} = W \times n \times \frac{D_1}{2}$$

$$381.96\times10^3 = 32\,l \times 6 \times \frac{132}{2}$$

$$l = 30.14\,mm \cong 32$$

$$W = 32\times30.14$$

$$\boxed{W = 964.48}\ N$$

$$\tau = \frac{W}{\frac{\pi}{4}d_1^2}$$

$$\tau = \frac{964.48}{\frac{\pi}{4}(\overset{20}{\cancel{8.1}})^2} = 3.07$$

$$M = W\left(\frac{l}{2} + 5\right)$$

$$= 964.48\left(\frac{\overset{32}{\cancel{30.14}}}{2} + 5\right)$$

$$= 20254.08\ N\text{-}mm$$

$$\sigma = \frac{M}{z} \qquad z = \frac{\pi}{32\,\cancel{d_1^3}}d_1^3$$

$$z = \frac{\pi\ (20^3)}{32\times\cancel{(20)}^3} = 1.227\times10^{-5}$$

$$\sigma = \frac{20254.08}{\underset{785.3}{\cancel{1.227\times10^{-5}}}} = 25.79$$

$$\text{Max. principle stress} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[25.79 + \sqrt{(25.79)^2 + 4(3.07)^2}\right]$$

$$= 25.9 \cong 30$$

Max. shear stress =

$$\frac{1}{2}\left[\sigma^2 + 4\tau^2\right]$$

$$= \frac{1}{2}\left[(25.79)^2 + 4(3.07)^2\right]$$

$$= 351.41$$

↓ Design of hub:-

$D = 2d$

$D = 2(40) = 80$

$l = 1.5d$

$l = (1.5)(40)$

$= 60$

$$T = \frac{\pi}{16} \frac{D^4 - d^4}{D} \cdot f_s$$

$$= \cancel{\frac{\pi}{4}} \frac{80^4 - 40^4}{\cancel{80}_{20}} \cdot \cancel{40}$$

$= 15079644.74$

Design of Key:

$$\omega = \frac{d}{4} = \frac{40}{4} = 10$$

$$T = f_s \times (l \times \omega) \times \frac{d}{2}$$

$$= 40 \times (32 \times 964.48) \times 20$$

$$= 24690688$$

$$\tau = f_c \times \left(l \times \frac{t}{2}\right) \times \frac{d}{2}$$

$= 80 \times (32 \times$